Lecture 17: Game Theory: An Introduction

1 Time Line

We have three more lectures and then a review. Leading up to our exam on Monday the 21st. I' will add a bunch of extra office hours next week. If you could please write down on a piece of paper what times would be good for you that would be great.

2 Game Theory

Game theory is the study of strategic behavior, or as Dixit and Nalebuff call it "the science of strategic thinking." At Michigan, you can take courses in game theory in economics, political science, the business school (sometimes), and the information school. Given that many of who have had game theory courses, we are not going to emphasize the particulars of game theory. Instead, we are going to look at it as a type of model and consider its strengths, weaknesses, and implications. We want to know when we should use a game theory model and when we should not, for while game theory is powerful, it is not the only way to model human social interaction.

In this first lecture on game theory, we will discuss two technical topics: *sequential games and backward induction* and *simultaneous games and equilibria*, and two broad questions *when are people strategic?* and *when do people optimize?*. We've already covered these two questions in an earlier lecture but I want to return to them. We begin with the questions.

3 When are people strategic?

Game theory relies on the assumption that people are strategic. But when will this be true and when won't it? Before we can even ask this question, we have to define what we mean by strategic. One definition of a strategic action is one that depends upon the actions of other people. But this definition doesn't quite work. Consider going into a grocery store. You take the prices as given. You are not strategic in what you choose. Yet your decisions depend upon the actions of other people because of the fact that they collectively determine the price. Therefore, the way that we are going to define strategic behavior is as behavior that takes into account the effect of others on you and of you on others. Both of these need not be present in a given situation, but the point is that your behavior is strategic if you contemplate both of those effects on an action.

So, when you get dressed in the morning, choose your lunch, decided which movie to go see you may not be being strategic. You may in fact be following simple rules that make you happy that do not take into account your effect on other people or other people's effect on you. However, when you choose a place to go study, if you are hoping to run into a particular person, then you may be making a strategic choice. Your decision depends upon the decision of someone else.

Another way to think about the difference between strategic behavior and non strategic behavior focuses on the number of actors. If there are lots of people shopping at Krogers, you benefit little by being strategic in what you buy. However, if you are playing tennis or poker, you have a huge benefit from acting strategically. Think of your decision about what major to choose. There is a sense in which this is a strategic decision. It depends upon what other do, but you really do not feel like this is strategic game with a few players. Therefore, you may just follow a simple rule. That rule may have strategic elements to it. For example, you may prefer to choose a rare major figuring that it will give you advantages. Thus, there isn't really a clean distinction between strategic and nonstrategic behavior. Instead, there are degrees of how strategically we behave. All else equal, the more strategically we behave, the better tool game theory becomes.

There are cases where people are not strategic.

Nice players: In some games, like the restaurant game that we will study below, I might want to be nice, not strategic. This can be placed in the larger context paradigm. I may be more concerned about having a good reputation. Of course it is possible to extend game theory so that people are being nice for strategic reasons. That is true. Some people are nice for those reasons. Other people are just nice. For example, the fact that people tip while traveling cannot be explained by reputation models.

Low stakes: If the payoffs don't matter to the players that much they may choose actions using other criteria. I am probably not strategic in what seat I choose when attending a lecture. Therefore, I follow a rule of thumb.

Cultural Constraints I once ran out of gas on the highway. A truck stopped within two minutes. He had gas in a can in the back of his car. He poured it in my tank and only asked that I pay to refill the tank at the next exit and hinted that he might need gas in his truck. Instead, he could have negotiated a price. I might have paid him $100 or so. It was winter. I was in Gary Indiana. It was dark. I had my neighbor's dog in the car because their dog sitter fell through. My neighbor's were 3 and 5 years old. I was going to be late seeing my family the day before thanksgiving. Had the person been strategic, he would have charged me much more. His decision probably had less to do with him being nice and non strategic than it had to do with culturally what type of exploitation we allow. Had I been in a foreign country, the norms might have been such that I would have had to pay all of my money.

It is also important that we draw a distinction between being strategic and optimizing. Game theory assumes that people are both strategic and that they are choose optimally given what other people do. There are many instances where we might expect people to be strategic but to not optimize.

Too many stages in game: In chess, even though there must be an equilibrium, we don't see it played. The game is too long to solve. The same is true of Go or even Monopoly. Therefore, I will be strategic but I will not optimize.

Payoff function too complicated: In choosing a major or a set of courses to take as an undergraduate, you are probably strategic but it is doubtful that you optimize (even given your information) because the problem is too hard.

Nested Games: Some actions belong to two games simultaneously. A vote on a bill is both part of a game within the legislature and part of a game with constituents or other candidates. It may not be possible to optimize in both games.

4 Sequential Games and Backward Induction

We now want to turn to some basic games. Consider the following simple game. Suppose that you and your brother are supposed to meet for dinner. I call this the restaurant game. You like sushi and he likes Italian food. You are suppose to call him at 6pm and tell him where to meet. If you choose a sushi restaurant, he'll reluctantly meet you there. If you choose an Italian restaurant, he'll also meet you but be much happier. If you are only concerned with your own happiness (and not your brother's), you will choose the sushi restaurant.

This is what Dixit and Nalebuff call Rule 1.

Rule 1: Look ahead and reason back.

They wrote this book with the intention of selling lots of copies (which they have!). The concept of looking ahead and reasoning back is formally called *backward induction*. We can write a full *game tree* for this game. We can then begin at the end of the tree and consider the optimal decision for each player at each node. At each subsequent node, we can then pick the best strategy for the next player. In using this logic, we get an *equilibrium* set of actions. This is a set of actions that no player would change given that they know the actions of the other players. It can be shown that any finite sequential game has an equilibrium that can be found this way.

Let's consider another example. There are two stacks, one containing one coin and one containing two coins. The rules of the game are as follows: In any one move a player can take as many coins as he or she likes from a single pile, but must remove at least one coin. The winner is the player who takes the last coin.

Suppose that you go first. We can write the current configuration of coins as (2,1). You have three possible actions. You can take two coins from the first pile, one coin from the first pile, or one coin from the second pile. Let's examine each:

If you take two coins from the first pile, you leave your opponent with the configuration (0,1), and the other player wins.

If you take one coin from the first pile, you leave your opponent with the configuration (1,1). At this point, the other player must and can only take one coin, leaving you with either the configuration (1,0) or (0,1). In either case, you win.

If you take one coin from the second pile, you leave your opponent with the configuration (2,0). At this point, the other player can take both coins from the first pile and win.

Therefore, your preferred strategy is to take one coin from the first pile. In any finite sequential game, you can use backward induction to solve it. The problem is that some games, like Go or Chess are too hard to solve that way. So people do not.

5 Simultaneous Games and Equilibria

In the two games we've considered so far, the players take actions sequentially. In many strategic situations, the agents take their actions at the same time. (Even if it is not at the exact same moment in time, so long as neither agent can base his or her action on the action of the other player.)

In the previous case, we could work backward in time and figure out the optimal action. Now, that's impossible. So, what we have to do is think hard about what the other person will do and what we should do.

Consider the following game. A candidate can decide either to tell the truth or to lie about a past transgression. The opponent can either emphasize the issue or ignore it. Suppose that even if the opponent ignores, that there is a good chance that the media will undertake an investigation. The payoffs (in utility terms) to the candidate may be as follows:

	emphasize	*ignore*
truth	5	4
lie	1	3

There isn't much to figure out here. Telling the truth is a dominant strategy. This is Rule 2 for Dixit and Nalebuff.

Rule 2: If you have a dominant strategy, use it

Now, let's look at this same game from the perspective of the opponent.

	emphasize	*ignore*
truth	2	3
lie	10	2

The opponent does not have a clear strategy. If the candidate is lying, then the opponent should emphasize the issue. But if the candidate is telling the truth the opponent should ignore the issue. If the opponent is strategic, he has an easy choice. The opponent should realize that the candidate has a dominant strategy: to tell the truth. Therefore, given that the opponent knows that the candidate will tell the truth, the opponent should ignore this issue. This is Dixit and Nalebuff's rule 3.

Rule 3: Eliminate any dominated strategies from consideration, and go on doing so successively

An implication of rule 3 is that a player who does not have a dominant strategy initially may get a dominant strategy after the other player's dominated strategies are removed from consideration.

5.1 So What?

So what do we do with this. There are three possible reactions to this. First, we could say, wow, let's construct lots of stylized mathematical examples and then chug and plug are way through these. That is what is done in a game theory class in an economics or math department. Second, we could do what Dixit and Nalebuff did and try to turn these sort of insights into basic thinking tools that we can apply to real situations in business, politics, and life. In doing so, we can *win*. Depending upon who you are that might seem like a good application.

We can loosely characterize these two approaches as mathematical chug and plug and intuitive application. Both are worthwhile. We're going to pursue a third approach that can be thought of as contemplative. We want to model situations involving people. Therefore, we want to ask, are people strategic? Do people optimize? and if not, why don't they. Depending upon the situation the answers to these questions can be easy or hard. For fun, here's another homework.

6 Homework

Take each of these four situations and answer whether you think people are strategic and whether they optimize. For each answer give two to five sentences (depending upon the subtlety of your reasoning) that explains your choices. This has two parts. First, plug in eight yes or no's, one in each box. Then defend each.

Situation	*Strategic?*	*Optimize?*
What people do on Friday nights		
Choice of Profession		
CDs people buy		
Allocation of time between studying and fun		