Emergence of Coordination in Scale-Free Networks

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Abstract

We use several models of scale-free graphs as underlying interaction graphs for a simple model of Multi-Agent Systems (MAS), and study how fast the system reaches a fixed-point, that is, the time it takes for the system to get a 90% of the agents in the same state. The interest of these kind of graphs is in the fact that the Internet, a very plausible environment for MAS, is a scale-free graph with high clustering and $< k_{nn} >$, the nearest neighbor average connectivity of nodes with connectivity k, following a power-law. Our results show that different types of scale-free graphs make the system as efficient as fully connected graphs, in a clear agreement with our previous research (*Artif. Intell.* **141**, pp. 175-181).

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1 Introduction

It has been long since Huberman & collaborators [15, 16, 17] showed that the study and/or design of a multiagent systems (MAS) must pay attention to the *dynamics* of the system. Important aspects of a MAS, such as performance, may depend on this dynamics. However, it looks like this topic has not attracted the attention of researchers in MAS [14, 33].

In this article we want to emphasize an important factor in the dynamics of a MAS: its topology. It is well-known that the topology of a MAS, that is, the pattern of interaction to be followed by the agents, may be highly determinant in what respects the long-time behavior of a dynamical system [1, 10, 12, 18, 23] and MAS are no exception. To do so, we will work in the framework introduced by Shoham & Tennenholtz [25, 26, 27, 28] of simple, game-theoretic agents trying to reach a social convention, that is, the same state for all agents (see below). We will not further discuss social conventions, see Shoham & Tennenholtz papers for details.

In the simplest multi-agent system (MAS) every agent may interact with every other agent. This means that the underlying topology is a graph with an all-to-all connectivity pattern. However, this is not very realistic, since MAS tend to run in an open environment, where agents have no knowledge about the whole. It is far more accurate to assume some restrictions in the pattern of interactions an agent may have. We can think of different possibilities: Regular graphs, lattices, etc. This has already been (partially) analysed, since emergence of conventions in MAS with topological restrictions has been studied in regular graphs (functional and product hierarchies, contract nets, decentralized and centralized markets, see [28] for definitions of these different types of graphs in the context of organization theory) and lattices [18, 19, 28, 29]. This work is quite interesting, since it shows that the underlying MAS topology is important in the efficiency of the emergence of conventions; however, regular topologies are not very realistic either. If we pay attention to the topology of *real* networks, we will find out that most of them have a very particular topology: they are *complex* networks [2, 4, 23, 6, 10, 12, 31] with nontrivial wiring schemes. Notice that one of the possible environments for a MAS, the Internet, is among the most prominent complex networks found in the real world. Complex networks are well characterized by some special properties, such as the connectivity distribution (either exponential or power-law) or the *small-world* property [23, 32].

In this article we will study the efficiency of the emergence of social conventions in MAS with a *scale-free* underlying topology, that is, its connectivity follows a power-law $P(k) \propto k^{-\gamma}$. We will follow the conceptual framework introduced by Shoham & Tennenholtz and our measure of efficiency will be one of those introduced in the work of Kittock [18]: the time it takes to reach a 90% of the agents in the system with the same state. This will be detailed below.

Previous experimental work by one of the authors [13] showed that a particular type of scale-free graphs (generated following the work of Barabási and collaborators [7, 3], see below) make the system behave in a way quite similar to the optimal underlying topology, that is, the complete graph. However, those graphs lack some properties that have been observed in the Internet, particularly, they lack a certain kind of correlations among nodes and the small-world property. In this article we continue that exploration by performing the same experimental work on different types of more sophisticated scale-free graphs. Readers of both papers (this one and [13]) will find some redundancy in the introductory part and the places where we describe the system which we used to perform the experiments, but this is unavoidable since this one is a sequel of [13].

2 Graph Models

As we pointed out in the introduction, recent discoveries on real networks lead us to think that regular and/or purely random graphs are not the most realistic environment for MAS. Lots of real networks have been studied [4, 5, 6, 8, 12, 23, 31] though the most interesting result for us is that the Internet is a *complex* network, a scale-free graph with small-world properties [2, 4]. Since the Internet is a quite reasonable environment for a MAS, and since the underlying topology is important for the efficiency of the emergence of conventions (as shown by Kittock [18]), it is quite clear that the study of the efficiency of the emergence of conventions in different types of scale-free networks will provide more realistic results.

Albert, Barabási and Jeong have recently proposed a set of different models for scale-free graphs, based on the growing process of the Internet and other real complex networks. We have used the Albert-Barabási [3] *extended* model as model of scale-free graphs, since it gives us some control over the exponent γ of the graph. The underlying idea is that of growth with preferential connectivity, where the most "popular" nodes get most of the links. This model was built on a simpler one [7], able to generate graphs with exponent $\gamma = 2.9 \pm 0.1$ (by setting $p = q = 0$ in the algorithm detailed below we recover this previous model). We will define precisely these graphs by giving an algorithm to build them.

The algorithm depends on 4 parameters: m_0 (initial number of nodes), m (number of links added and/or rewired at every step of the algorithm), p (probability of adding links) and q (probability of edge rewiring). The procedure is: Start the algorithm with m_0 isolated nodes, and perform at every step one of these three actions:

1 With probability p add m ($\leq m_0$) new links. We pick two nodes randomly. The starting point of the link is chosen uniformly and the end point of the new link will be chosen according to the following probability distribution:

$$\Pi_i = \frac{k_i + 1}{\sum_j (k_j + 1)}$$

where Π_i is the probability of selecting the i-th node, and k_i is the number of edges of node i. This process is repeated m times.

2 With probability q, m edges are rewired. That is, we repeat m times: Choose (uniformly) at random one node i and a link l_{ij}. Delete this link. Choose another (different) node k with probability $\{\Pi_l\}_{l=1 \ldots N}$ and add the new link l_{ik}.

3 With probability $1-p-q$ add a new node with m links. These new links will connect the new node to m other nodes chosen according to $\{\Pi_l\}_{l=1...N}$.

Once we get the desired number N of nodes, we stop the algorithm. The graphs generated with this algorithm are scale-free *random* graphs, that is, there are no correlations among edges [24]. It can be shown that in the limit of large N, when $p = q$, this algorithm ends up with a graph with connectivity distribution

$$P(k) \propto (k+1)^{-\left(\frac{2m(1-p)+1-2p}{m}+1\right)}$$

that can be approximated, when $k >> 1$, by $P(k) \propto k^{-\gamma}$ where $\gamma = \frac{2m(1-p)+1-2p}{m} + 1$. The graphs in our experiments are no larger than $N = 10^5$, therefore the theoretical exponent suffers from finite-size effects and must be computed numerically. (see figure 1).

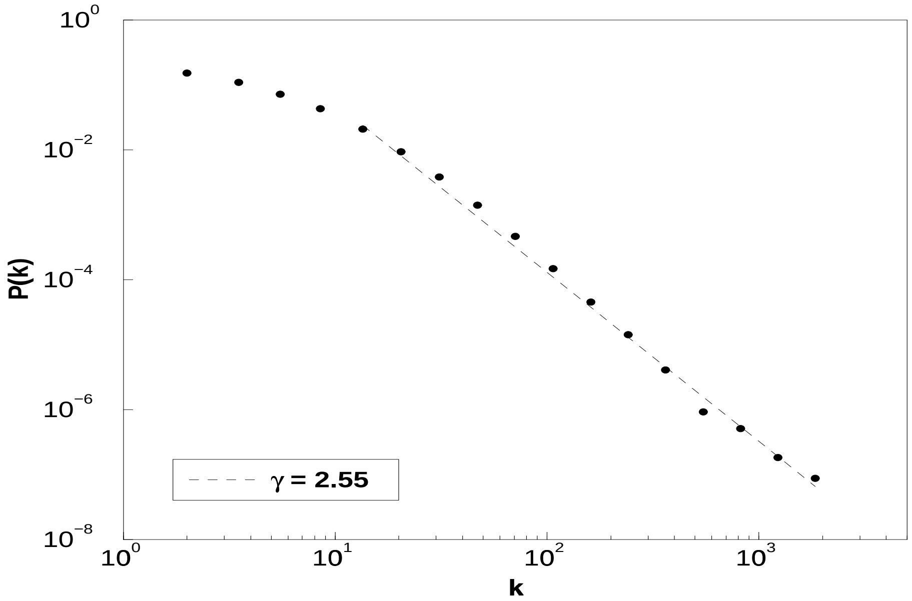

Figure 1: This figure shows the connectivity distribution of a graph generated with the Albert-Barabási extended model (see text). Parameters are $N = 5 \times 10^4$, $p = q = 0.4$, $m_0 = 4$ and $m = 2$, so the exponent should be $\gamma = 2.3$. As we see in the plot, the real exponent is $\gamma \sim 2.5$. The data were logarithmically binned.

Albert-Barabási's scale-free graphs are simple models that lack some of the characteristics of the Internet. Pastor-Satorras et al. [24] made some measures on real samples of the Internet, showing a large clustering, an exponent $\gamma \sim 2.25$ and the presence of correlations in the form of a power-law dependence in the nearest neighbors average connectivity of nodes with connectivity k (quantity they called $< k_{nn} >$). Albert-Barabási graphs have low clustering and no correlations, so we will work with two more models of scale-free graphs.

First, the Walsh model of scale-free graphs with high degree [30]. This model is a variation of the Albert-Barabási model we have seen, but with parameters $p = q = 0$ and a rule to connect a new node that reads as $min(1, mk_i / \sum_i k_i)$. Graphs generated with the Albert-Barabási model, due to the probability distribution they use to model preferential attachment, differ from the Internet, among other things, in that the clustering of the Internet is much higher than the clustering of these models. Walsh modification is able to induce high density graphs, with large clustering, maintaining the scale free property with an exponent $\gamma \simeq 2.9$.

Second, the Bianconi-Barabási model [8], very similar to the Albert-Barabási model (also with $p = q = 0$) but introducing a parameter called *fitness*, associated to every node. This new parameter is introduced to account for the evidence that evolving systems with a scale-free topology are often *competitive* systems: not all nodes are equally successful in acquiring links. Thus, they define the new rule of connectivity as

$$\Pi_i = \eta_i k_i / \sum_j \eta_j k_j$$

for every node $i = 1 \ldots N$. In our case, we have chosen η_i uniformly from the unit interval. These graphs are *generalized* scale-free graphs, since its distribution follows $P(k) \propto k^{-\gamma} / \log(k)$ for $\gamma \simeq 2.25$ (an exponent very similar to that found for the Internet [24]). These graphs are interesting since their correlation $< k_{nn} >$ scales following a power-law, as does the Internet [24]. Let S_k be the set of nodes with k neighbors, N_k the cardinal of S_k, V_j the set of neighbors of node j and c_i the number of neighbors of node i. We define $< k_{nn} >$:

$$< k_{nn} >= \frac{1}{N_k} \sum_{j \in S_k} \frac{1}{k} \sum_{i \in V_j} c_i$$

We can see in figure 2, $< k_{nn} >$ measured for our three models of scale-free graphs. We see that only the fitness model has correlations similar to the ones measured in the Internet [24].

Thus, we will work with scale-free graphs of the Albert-Barabási type, and two variations: the Walsh type, with a higher clustering and the Bianconi-Barabási (fitness) type, with $< k_{nn} >$ following a power-law. The Walsh model and the Albert-Barabási model have no correlations at all. However, only the Walsh model has high clustering.

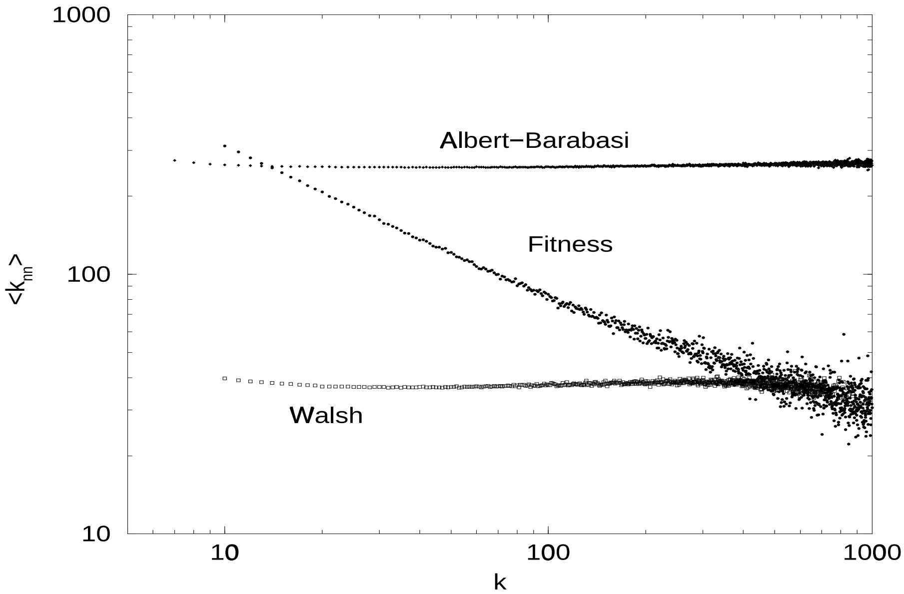

Figure 2: This figure shows $< k_{nn} >$ for our three models of scale-free graphs. Parameters are: $N = 10^4$, $m_0 = 9$ and $m = 8$ for all graphs. Besides, $p = q = 0.4$ for the Albert-Barabási model. We observe no correlations for the Walsh model and the Albert-Barabási model.

3 Social Conventions in MAS

Our MAS will consist of N agents on a graph, where every agent will be located on a node of the graph. Its adjacent agents will be called its *neighbors*. Every agent will be in one out of two states (or actions), called A and B. The system will evolve in time, and at each time step one agent will be selected at random, for state updating. The dynamics we use is *asynchronous*, following previous work [18, 19, 27]. We will depart from Walker & Wooldridge [29] formalization because the dynamics they use imposes a *synchronous* dynamics, where all agents interact at once, though it is well known that some "emergent" properties of synchronous systems are not due to the system itself, but to global correlations introduced by this synchronous update [16, 21]. Different rules to update agent's state will define different systems. As in [13], we will study the *highest current reward* (HCR) rule and the *generalized simple majority* (GSM) rule.

We define a *social convention* as in [27]: A social law is a restriction on the set of actions available to agents. A social law that restricts the agents' behavior to one particular action is called a social convention. In our case a social convention will be reached if all the N agents are either in state A or in state B.

From [18] we will get the performance measure we use to evaluate how fast conventions arise in our systems, it is the *convergence time* T_c: the convergence time for a given level of convergence c is the earliest time at which $C_t \geq c$, where C_t is the convergence of a system at time t, that is, the fraction of agents using the majority action (either A or B). In this note we will focus on the study of the average time to a fixed convergence (we set c to 90%, following [18]).

3.1 The Generalized Simple Majority rule

This rule was introduced in our previous work [13]. Our departure point is the simple majority rule, as defined by Walker & Wooldridge [29], generalized in such a way that now the amount of neighbors in a certain state does not determine a change of state, but this change is stochastic.

Let us assume that a node i is in a state $S \in \{A, B\}$ and has k neighbors. Let $\overline{S}$ be the complementary state and $k_{\overline{S}}$ the number of neighbors in state $\overline{S}$. Then, agent i will change to state $\overline{(S)}$ with probability:

$$f_\beta(k_{\overline{S}}) = \frac{1}{1 + e^{2\beta(2k_{\overline{S}}/k - 1)}}$$

This rule generalizes simple majority since for $\beta \to \infty$ we recover the change of state only when more than $k/2$ neighbors are in state $\overline{S}$.

There is no theorem assuring convergence in the emergence of conventions in the system defined with the GSM rule, but we provided some analytical evidence that this is the case [13] (in this paper we set $\beta = 10$).

3.2 The Highest Current Reward rule

The framework in which we will work here was introduced by Shoham & Tennenholtz [25, 26, 27, 28] some time ago, though it is in frequent use nowadays (see [9, 11] for example). It is much more sophisticated than the previous one (the GSM rule), so it will need some detailed explanations. In this note we will adapt from Shoham & Tennenholtz [27] the definitions and theorems we need, not dwelling on justifications of this formal framework (it was eloquently done there). We will focus on *coordination* games [20] A payoff matrix G 2×2 defines a 2-person 2-choice symmetric coordination game if G has the form

$$\begin{pmatrix} x & u \\ v & y \end{pmatrix}$$

where $x > v$ and $y > u$.

Essentially the idea is that every player has two available actions, say A and B. If both players play A, both players receive a payoff of x. If they play B they receive a payoff of y. When the players do not agree, for example, player 1 plays A and player 2 plays B, the former receives a payoff of u and the latter a payoff of v; the remaining situation is symmetric. The condition on the entries of G makes clear that to play the same action is the best choice. Specifically, we will use the *pure coordination game* [20] G, where $x = y = +1$ and $u = v = -1$.

Now, once defined the game we need to define the players. Our MAS will be composed of N agents (every agent is a player) that will interact with other agents, playing the game G once per interaction. What we are interested in is whether the dynamics of this system makes all the agents reach a social convention. In our particular setting, this means that we want to know whether all the agents will end up playing one of the two possible actions of the game G, say A and B.

Following [18], every agent, say the k-th, will be characterized by a *memory* M_k of size M (same size for all the agents) and an action a_k (to play the next time agent k is selected, so the value of a_k is either A or B). The memory M_k will record some information on the M last plays of the agent k: The value of the position i of the memory M_k will be a tuple $< a_k^i, p_k^i, t^i >$ where t^i is the time the i-th play took place, a_k^i is the action played by agent k and p_k^i is the payoff received ($1 \leq i \leq M$). However, in this work we will not study the effect of memory, setting $M = 1$.

Now we must define the dynamics of the system (a variant of $n-k-g$ stochastic social games [27] where we will take into account the underlying topology). At every time step t, a pair of agents will be selected to play the game G, where one of them will be randomly chosen and the other will be one of its neighbors, according to the underlying graph. They will receive a payoff (either $+1$ or -1) depending on their actions. Let us assume that at time t, agents k (with memory M_k and action a_k) and l (with memory M_l and action a_l) are chosen to play. Every agent will receive a certain payoff, say p_k and p_l. Now, agent k must decide which action it is going to play next time it is chosen, as a function of its memory M_k, the action a_k played and the payoff received p_k. It uses the *Highest Current Reward* rule. Agent k will compute the payoff received for using action A in the last M plays in which it has been involved: $P_A^k = \sum_{i:a_k^i=A} p_k^i$, where P_B^k is defined in the same way. Agent k will add p_k to either P_A^k or P_B^k, depending on a_k. Now, agent k can decide: Next time it is chosen to play, the action chosen by the agent k will be either A if $P_A^k > P_B^k$, B if $P_B^k > P_A^k$ or a_k otherwise. Finally agent k updates its memory, deleting the oldest entry and adding the tuple $< a_k, p_k, t >$ (agent l will do the same thing, the rest of the system will do nothing). Shoham & Tennenholtz [26, 27] provide a general theorem that guarantees the convergence of this system to a stable social convention.

Kittock [18] studied numerically the efficiency of the emergence of conventions in regular graphs. His main result was that the underlying topology has a profound effect on the efficiency with which conventions emerge, and he conjectured that this efficiency depends essentially on the *diameter* of the graph. So, the optimal graph will be the graph with an all-to-all connectivity pattern (which we will call K_N).

4 Results: $T_{90\%}$ vs. N

Once we know that social conventions will emerge in the systems we are interested in, we would like to know how fast these conventions will be reached.

The results of our experiments can be seen in figure 3 and figure 4. We performed experiments analogous to those of [13]. We measured $T_{90\%}$ vs. N in systems using the HCR rule or the GSM rule and our three models of scale-free graphs: Albert-Barabási models, Walsh model and Fitness model. For every N, we ran 25 simulations of the system initialized randomly (agents with random initial state, either A or B with probability 0.5), averaging the results. See table for graph parameters.

Fig 3 & 4	Graph Parameters
K_N	None needed
$S_N^{2.72}$	$m_0 = 4$, $m = 1$, $p = q = 0.4$
$S_N^{2.55}$	$m_0 = 4$, $m = 2$, $p = q = 0.4$
$S_N^{2.52}$	$m_0 = 6$, $m = 3$, $p = q = 0.4$
S_N Walsh	$m_0 = 6$, $m = 4$
S_N Fitness	$m = 4$

From our numerical work (these figures are representative of results obtained with different sets of parameters) we observe that $T_{90\%}^{HCR} = O(N \log N)$ and $T_{90\%}^{GSM} = O(N)$ for scale-free graphs. In the case of the HCR rule this fact is remarkable, since this $O(N \log N)$ behavior is not found in regular graphs (that follow a $O(N^3)$ law [18, 13]) and it is the lower bound predicted analytically in [27].

In agreement with our previous results [13], the growth of $T_{90\%}$ is identical (asymptotically) to the optimal growth of $O(N \log N)$ (HCR rule) and $O(N)$ (GSM rule) observed in the system with the K_N graph as underlying topology. This case, provided Kittock conjecture is correct (the growth depends on the diameter of the graph [18]), is the optimal case. This identical behavior

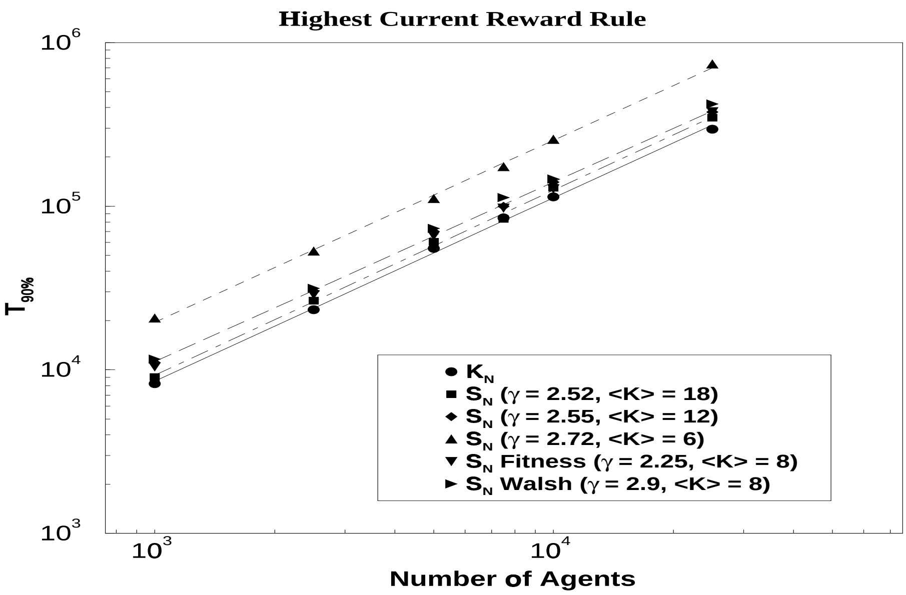

Figure 3: HCR rule: $T_{90\%}$ vs. N, averaged over 25 samples for each N. Several graphs have been used (see text and table for parameters; Sizes: $N = 1, 2.5, 5, 7.5, 10, 25 \times 10^3$).

is obtained with graphs whose *connectivity* is much smaller than $N-1$ (corresponding to graphs K_N), so that we have a topology that makes the system more efficient at a lower cost (where the cost would be a measure of the structure of the graph, in this case, the connectivity).

However, in [13] we put forward the hypothesis that what was really important in accounting for the growth of $T_{90\%}$ observed in our simulations was the *randomness* of the graph, that is, the absence of any kind of correlations. The main motivation of this work has been to verify this hypothesis by using graphs with the scale-free property but some added structure. Our results seem to point out that the phenomenon observed is somewhat more complicated than we conjectured, since we observe an almost identical behavior using either the Albert-Barabási extended model or the Walsh model (with constant $< k_{nn} >$) and the Fitness model (with $< k_{nn} >$ following a power-law). The data obtained would support the hypothesis that merely the scale-free property is enough to account for the observed results. Note that this remark does not affect the fact that our results are consistent with Kittock conjecture [18], that is, the growth of the

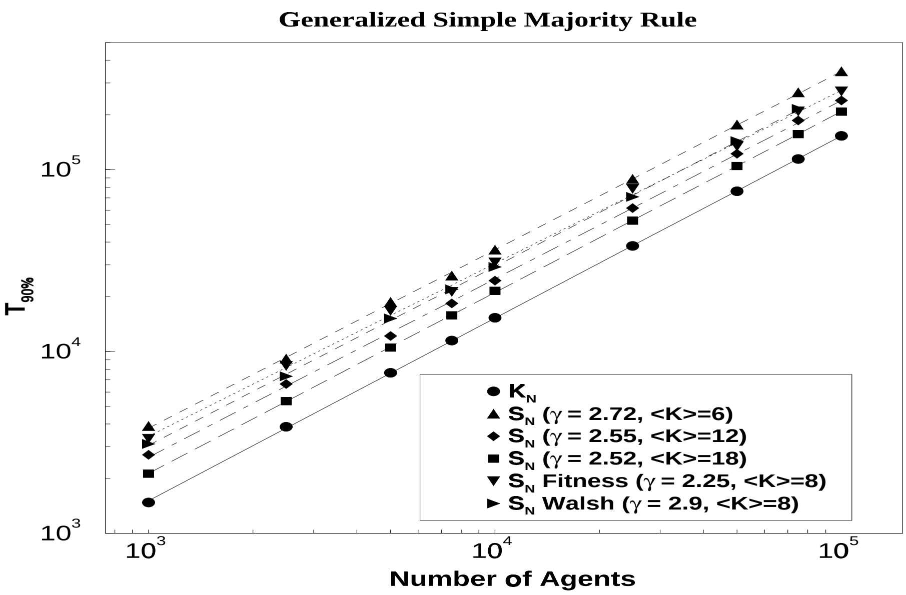

Figure 4: GSM rule: $T_{90\%}$ vs. N, averaged over 25 samples for each N. Several graphs have been used (see text and table for parameters; Sizes: $N = 1, 2.5, 5, 7.5, 10, 25, 50, 75, 100 \times 10^3$).

efficiency of the emergence of social conventions depends mainly on the diameter of the graph. The question here is that the growth of the diameter of the scale-free graphs we used is still logarithmic, despite the added structure they may have.

Finally, we do not mention the slight differences between the results found with the particular graphs we used (see figures 3 and 4) since we have used graphs of different average connectivities, which is important in relation to the diameter of the graph. The relevant information, discussed above, is the growth law of $T_{90\%}$ vs. N.

5 Summary and Prospects

In this paper we continued the work initiated in [13], investigating the efficiency of the emergence of social conventions in simple, game-theoretic multi-agent systems with realistic underlying topologies. Our results are in complete agreement with the results obtained in [13].

Kittock conjecture [18] that the growth of the efficiency of the emergence of social conventions depends mainly on the diameter of the graph is consistent with our results. However, the importance we attributed to randomness in [13] looks now less relevant, in favour of whether the graph is or is not scale-free.

There are some immediate ways to continue with our investigation: we could measure $T_{90\%}$ depending on the memory of the agents (for the HCR rule) or the uncertainty of state changes (different β in the GSM rule), we could repeat the same work with *cooperation* games [1, 18, 27].

Further work may also be done with mobile agents, since systems with agents that are able to move may display complex behavior [22].

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