

PERISHABLE DURABLE GOODS

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ABSTRACT. We examine whether the Coase conjecture [7, 12, 4, 10] is robust against slight ability of commitment of the monopolist not to sell the durable goods to consumers with a reservation value higher than the marginal production cost. We quantify the commitment ability in terms of the speed that the durable goods perish while in the hand of the monopolist, because it better captures the notion of irreversibility implied by the commitment than the time interval between the offers. We demonstrate that the slight commitment capability makes a substantial difference in the equilibrium outcomes by constructing two kinds of reservation price equilibria [10] that refute the Coase conjecture. In the first equilibrium, the monopolist can credibly delay to make an acceptable offer. Almost all consumers are served, but only after very long delay. As a result, the total gains from trading is arbitrarily small. In the second equilibrium, the monopolist's expected profit can be made close to the static monopoly profit, if the goods perish very slowly. This result differs from [3] where the good is depreciated after being delivered to the consumers, because the difference from the competitive equilibrium outcome does not vanish even if the time between the offers and the rate of decay converge to 0. Various extensions are discussed.

1. INTRODUCTION

The Coase conjecture [7, 4, 12, 10] shows that the dynamic foundation of the monopolistic power lies in the monopolist's commitment ability not to sell durable goods to consumers who are willing to pay more than the marginal production cost. By commitment, we mean an action that entails an irreversible consequence. Perishability captures the irreversibility in terms of quantity while the time between the offers measure the irreversibility in terms of timing of sales which is generally considered a measure for the commitment ability of the monopolist. In order to highlight the different aspect of commitment, we focus on a model with perishable durable goods, where the time between the offers is small.

We quantify the monopolist's commitment ability in terms of the speed that the durable goods perish away, while focusing on the case with a short interval between the offers. We examine the dynamic monopolistic market for durable goods, which perish at the instant rate of e^{-b} for some small $b > 0$.¹

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¹If $b = \infty$, it is the case of complete commitment, while $b = 0$ corresponds to the classic durable good monopoly problem.

Despite an extensive literature on the dynamic monopoly problem as well as sequential bargaining models (e.g., [11, 9, 2, 5, 6, 8]), we have little understanding about how the market outcome changes with respect to the ability of commitment of the monopolist, except for the two limit cases: complete commitment and no commitment. In order to examine how robust the Coase conjecture is, we should examine whether the Coase conjecture continues to hold, if the monopolist can be committed not to sell a small amount of goods to consumers.

We differentiate “perish” from “depreciate,” while we use “perish,” “decay” and “burn off” interchangeably. Perishable goods decay before they are sold, which affects the future supply of the goods irreversibly. On the other hand, goods depreciate only after they are delivered to consumers, which generate the demand for replacement. The strategic impact of replacement purchases was analyzed by [3]. It is shown that with a positive rate of depreciation, the monopolist is not willing to provide the competitive level of goods. Yet, the gap between the competitive outcome and the monopolistic outcome vanishes as the time between offers and the depreciation factor converge to 0. In order to highlight the impact of slight decay, let us assume that the good is not depreciated once it is delivered to consumers.

In contrast to [3], we show a significant discontinuity in outcomes with respect to the rate of decay around $b = 0$ (no decay). To differentiate two cases, we call the durable good problem with $b = 0$ (no decay) *classic problem*, while the case with $b > 0$ (decay) is referred to as *perishable problem*. To highlight the impact of the slight decay, we focus on the same rule of game as the classic durable goods monopoly problem with the linear (inverse) market demand curve $p = 1 - q$ where the monopolist offer p_t in period t , which was accepted or rejected by consumers. After the offer is rejected, the monopolist has to wait for $\Delta > 0$ before offering p_{t+1} . The game continues until either all consumers are served, or all available stock is sold. All agents are risk neutral with the same discount factor $\delta = e^{-r\Delta}$ for some $r > 0$.

We choose to focus on the case where the demand curve does not hit $p = 0$, known as “gap case” in order to sharpen the comparison. In this case, the classic problem has a unique subgame perfect equilibrium where the consumer’s acceptance rule can be represented as a threshold rule. We call such a subgame perfect equilibrium a reservation price equilibrium [10], for which the Coase conjecture holds: in any reservation price equilibrium, the initial offer converges to the lowest reservation value of the consumers, and all consumers are served almost immediately as $\Delta \rightarrow 0$.

We construct two reservation price equilibria when $q_f < 1$, which roughly forms the upper and the lower bounds of the set of all reservation price equilibria. In the first equilibrium, the monopolist’s expected profit is close to 0 as $q_f < 1$ is close to 1 and $b \rightarrow 0$. Interestingly, almost all consumers are served but the market outcome is extremely inefficient. The monopolist credibly delays to make an acceptable offer until the available stock reaches the target level. Because $b > 0$ is small, it takes exceedingly long periods to reach the level, and the consumer surplus is discounted away. While the monopolist generates profit slightly higher than what he could have made in the equilibrium satisfying the Coase conjecture, his profit is also very small.

In the second reservation price equilibrium, the monopolist’s expected profit is close to what he can achieve in the static monopoly, if both $q_f < 1$ and $y > 1$ are close to 1, and

$b > 0$ is close to 0. The slow decay opens up a strategic opportunity for the monopolist to credibly delay to make an acceptable offer for a significant time. If the consumer knows that an acceptable offer will arrive in the distant future, he is willing to accept the high price on the table. By exploiting the impatience of the consumers, the monopolist can achieve almost the maximum profit among all incentive compatible mechanism.

The first equilibrium can be served as a credible threat against any deviation by the monopolist. Following the idea of constructing non-stationary equilibria of [2], we can sustain any level of monopolist's profit as a subgame perfect equilibrium.

The set of subgame perfect equilibria is discontinuous with respect to the perishability of the durable goods in a number of important ways. First, the subgame perfect equilibrium that satisfies the Coase conjecture is no longer an equilibrium in some nearby game to the classic problem. Second, the equilibrium outcome in the nearby game is much richer than the classic problem. Thus, the equilibrium analysis does provide us a precise benchmark, against which the actual market outcome can be compared to. These two features of equilibrium outcomes have an important policy implication that a substantial market power does not necessarily imply substantial commitment power. Thus, the classical remedy to unravel the commitment capability of the monopolist not be as effective as the classic problem suggests, because it is practically impossible to completely eliminate the commitment power.

The rest of the paper is organized as follows. Section 2 formally describes the model and examines the key results from the classic durable good monopoly problem. Section 3 examines a simple example where the market demand curve is a step function. Although our main result is built around a linear demand curve, we begin with this example, because we can precisely calculate the subgame perfect equilibrium to reveal the key properties of the equilibrium we will analyze. Section 4 analyze a market with a linear demand curve. In Section 4.1, we explore an artificial game in order to highlight the mechanism that prompts the monopolist to delay to make acceptable offers. We calculate an equilibrium of the game. In Section 4.2, we construct a reservation price equilibrium, which approximates the equilibrium of the artificial game in Section 4.1. We observe that the monopolist may spend many periods without making acceptable offers, while waiting for the available stock to be perished to reach the target level. The equilibrium constructed in Section 4 seems to indicate that the monopolist's profit should be small, if the monopolist has little ability to commit himself not to sell (small $\Delta > 0$ and $b > 0$). Section 5 shows otherwise by constructing an equilibrium where the monopolist can generate a large profit. Section 5.1 examines another artificial game, in which the monopolist can choose the time interval of making unacceptable offers to highlight the structure of the second equilibrium. We show that as $b \rightarrow 0$, the monopolist's equilibrium in this game converges to the static monopoly profit. In Section 5.2, we construct a reservation price equilibrium, which approximate the equilibrium constructed in Section 5.1. Section 6 concludes the paper with discussions on extensions and policy implications.

2. PRELIMINARIES

Except for Section 3, we focus on a market where the demand curve is linear:

$$(2.1) \quad p = 1 - q \quad q \leq q_f < q$$

where $q \in [0, q_f]$ is the size of the consumers who were served, and p is the delivery price. We regard each point in $[0, q_f]$ as an individual consumer. By consumer q , we mean a consumer whose reservation value is $1 - q$. We call q_f the size of the whole market.

We write a residual demand curve as $D(q_0, q_f)$ after $q_0 \in [0, q_f]$ consumers are served. Let y_t be the amount of stock available at the beginning of period t . Except for Section 3, we assume that the initial stock is sufficient to meet all demand in the market: $y = y_1 > q_f$.

Let q_t be the total mass of consumers who has been served by the end of period t . Thus, $q_t - q_{t-1}$ is the amount of sales in period t . Then,

$$y_{t+1} = \beta(y_t - (q_t - q_{t-1}))$$

for $\beta = e^{-\Delta b}$, $\Delta > 0$ and $b > 0$. We call $\Delta > 0$ the time interval between the offers, and $b > 0$ the instantaneous rate of decay.

Let h_t be the history at period t , that is a sequence of previous offers (p_1, \dots, p_{t-1}) . A strategy of the monopolist is a sequence $\sigma = (\sigma_1, \dots, \sigma_t, \dots)$ where $\sigma_t(h_t) = p_t \in \mathbb{R}_+$ $\forall t \geq 1$. Let Σ be the set of strategies of the monopolist. Similarly, a strategy of a consumer q is a mapping from his reservation value $1 - q$, history of offers and the present offer p to a decision to accept or reject. If he purchases the good at p , then his surplus is $(1 - q) - p$. All agents in the model have the same discount factor $\delta = e^{-r\Delta}$ for $r > 0$.

Let $\{q_0, q_1, q_2, \dots, q_t, \dots\}$ be a sequence of *weakly* increasing numbers, which represent the sequence of the total mass of consumers who have been served by the end of period t . Naturally, $q_0 = 0$. Let \mathbf{Q} be the set of all such sequences. The monopolist's profit is

$$\sum_{t=1}^{\infty} \delta^{t-1} (q_t - q_{t-1}) p_t$$

where $p_t = \sigma_t(h_t)$ where $h_t = (p_1, \dots, p_{t-1})$.

We say that the market is cleared at t if the monopolist meets all the demand $q_t = q_f$ or sells all remaining stock $q_t - q_{t-1} = y_t > 0$ for the first time at $t < \infty$. We know that in the classic problem, the market is cleared in a finite number of periods if $q_f < 1$ [9, 10].

Given the monopolist's strategy σ , consumer q 's action is optimal if he accepts p_t in period t if

$$(1 - q) - \sigma_t(p_1, \dots, p_{t-1}) > \sup_{k \geq 1} \delta^k ((1 - q) - \sigma_{t+k}(p_1, \dots, p_{t-1}, p_t, \dots, p_{t+k-1}))$$

and rejects, if the inequality is reversed, where p_t is realized according to σ , $\forall t \geq 1$. By exploiting the monotonicity with respect to the reservation value, the classic problem allows us to write the optimality condition of the consumers more compactly by focusing on the *critical type* $1 - q_t$, who is indifferent between accepting the present offer and the next offer:

$$(1 - q_t) - p_t = \delta ((1 - q_t) - p_{t+1})$$

where $p_t = \sigma_t(h_t)$ and $h_t = (p_1, \dots, p_{t-1})$. However, in the perishable problem, we have to admit the possibility that $q_t = q_{t-1}$ for some $t \geq 1$ if the monopolist makes unacceptable offers.² We need to write the consumer's optimality condition in a more general way:

$$(2.2) \quad (1 - q_t) - p_t = \sup_{k \geq 1} \delta^k ((1 - q_t) - p_{t+k}).$$

²Actually, he does in an equilibrium.

We can define a Nash equilibrium in terms of the monopolist's strategy σ that solves

$$\max_{\sigma \in \Sigma} \sum_{t=1}^{\infty} \delta^{t-1} (q_{t-1} - q_t) p_t$$

where $p_t = \sigma_t(h_t)$ where $\sigma = (\sigma_1, \sigma_2, \dots)$ and $h_t = (p_1, \dots, p_{t-1})$ and $\mathbf{q} = (q_1, q_2, \dots)$ satisfies (2.2). We say that σ is a subgame perfect equilibrium if σ induces a Nash equilibrium following every history. We shall focus on a class of subgame perfect equilibria where a consumer's strategy is characterized by a threshold rule, which is a natural state variable of the game, namely the residual demand and the available stock.

Definition 2.1. *A subgame perfect equilibrium is a reservation price equilibrium, if there exists $P : [0, q_f]^2 \times [0, y] \rightarrow \mathbb{R}$ such that*

$$p_t = P(q_t, q_f, y_t)$$

with $y_t = \beta(y_{t-1} - (q_t - q_{t-1}))$ where p_t is the equilibrium price offered in period t , q_t is the total mass of consumers served by the end of period t and y_t is the available stock at the beginning of period t .

The Coase conjecture holds in a reservation price equilibrium for the classic durable good problem.

Theorem 2.2. [12, 4, 10] *Suppose that $b = 0$. A (generically) unique reservation price equilibrium exists if $q_f < 1$. In any reservation price equilibrium, the initial offer of the monopolist converges to $1 - q_f$ as $\Delta \rightarrow 0$.*

Before describing and analyzing the perishable problem, let us describe the classic problem to review the key results. The optimization problem of the risk neutral monopolist for the classic problem is to choose a sequence $\mathbf{q} \in \mathbf{Q}$ to maximize the discounted expected profit subject to a couple of constraints:

$$(2.3) \quad \mathcal{V}^c(q_0, q_f, y) = \max_{\mathbf{q} \in \mathbf{Q}} \sum_{t=1}^{\infty} p_t (q_t - q_{t-1}) \delta^{t-1}$$

$$(2.4) \quad \text{subject to} \quad (1 - q_t) - p_t = \delta((1 - q_t) - p_{t+1})$$

$$(2.5) \quad \lim_{t \rightarrow \infty} (1 - q_t) - p_t = 0.$$

(2.4) is the constraint imposed by the rational expectations of the consumers, which renders p_t as a function of q_t and q_{t+1} . (2.5) implies that in order to clear the market, the "final" offer of the monopolist must be the lowest reservation value of the consumer. If $q_f = 1$ so that the lowest reservation value is 0, then the market opens indefinitely so that there is no "final" offer. Yet, the price must converge to 0 as $t \rightarrow \infty$.

Let $T_f(q_0, q_f, y)$ be the total number of periods needed to make sales before serving every consumer in the market, or exhausting all available stock. If it takes infinitely many periods to serve all consumers, we let $T_f(q_0, q_f, y) = \infty$. In the classic problem, $T_f(q_0, q_f, y)$ is precisely the number of offers the monopolist makes in the game. Since the monopolist must sell with a positive probability in any period in the equilibrium strategy,

$$q_t < q_{t+1} \quad \forall t \geq 1$$

unless the market is closed in period t . We can understand T_f as the last period of the sales in the classic problem. For the later analysis, it would be more convenient to interpret T_f as the first period to clear the market either by serving all consumers ($q_t = q_f$) or exhausting all remaining stock ($q_t - q_{t-1} = y_t$):

$$T_f = \inf\{t : q_t = \min(q_f, y_t + q_{t-1})\}.$$

Let us summarize the key properties of the subgame perfect equilibrium in the classic problem, which will be a key building block for constructing a reservation price equilibrium in the perishable problem.

- Lemma 2.3.** (1) *If $\forall y, q_f \neq q'_f$, then there is no $q \in [0, q_f]$ such that $P(q, q_f, y) = P(q, q'_f, y)$.*
 (2) *If $q_f < 1$, $T_f(q_0, q_f, y) < \infty$.*
 (3) *$\mathbf{q}(q_0, q_f, y)$ is a continuous function of (q_0, q_f, y) if $q_f < 1$.*
 (4) *$T_f(q_0, q_f, y)$ is a decreasing function of q_0 , but increasing function of q_f .*
 (5) *Given an optimal sequence $\mathbf{q}(q_0, q_f, y) = \{q_t\}$ and associated $T_f(q_0, q_f, y)$, define*

$$y_f(q_0, q_f, y) = \sum_{t=1}^{T_f(q_0, q_f, y)} (q_t - q_{t-1})\beta^{-t}$$

Then, $y_f(q_0, q_f, y)$ is a continuous function of (q_0, q_f, y) .

- (6) *$y_f(q_0, q_f, y)$ is a strictly increasing function of q_f .*

Proof. See Appendix A. □

3. TWO TYPES

Before plunging into formal analysis of the perishable problem, let us consider a simple example, where we can calculate the subgame perfect equilibrium. Instead of a market with a linear demand, we consider a market populated with a continuum of infinitesimal consumers, whose total mass is $x + \frac{1}{2}$. x units of the consumers have valuation 3 and the remaining consumers have valuation 1. We assume that $x \in (0, 0.5]$. The monopolist has y amount of perishable but durable goods. Assume that $y \geq 0.5$ so that in the initial period, the monopolist can serve every high valuation consumer. The market opens in period $t = 1, 2, 3, \dots$ until every consumer is served, or all goods are sold, whichever comes first. A consumer demand precisely a single unit and never comes back to the market after the purchase. Following [10], we assume that two different residual demand curves which differ only over the null set of consumers induce the same continuation strategy of the monopolist.

Let us focus on the case where

$$0 \leq x \leq y \leq x + \frac{1}{2}.$$

If $x > y$, then the monopolist can charge 3 credibly to serve every high valuation buyer and close the market. If $y \leq x + \frac{1}{2}$, the available stock is less than the total number of remaining consumers, including the low valuation buyers. We shall discuss the remaining cases after we completely analyze the most interesting cases.

3.1. Construction of an Equilibrium. In the classic problem ($b = 0$), the optimal pricing rule is to open the market for two periods, offering $p_1 = 3 - 2\delta$ and $p_2 = 1$ unless x is too small. The initial offer will be accepted by the high valuation consumers, while the last offer serves all remaining low valuation consumers.

Suppose that the goods decay ($b > 0$). What would be the initial offer from the monopolist? The answer depends upon how quickly $y_t \leq x$, for a given level of patience of the players. For example, if $y_2 = \beta y \leq x$, then the monopolist can credibly charge 3 from the initial period, which will be accepted by all high valuation consumers. What if y decays slowly so that $y_3 = \beta^2 y \leq x < \beta y$? The answer to the same question is no longer obvious. In fact, if the monopolist is sufficiently patient (small $r > 0$), he will find it optimal not to make any sales in the initial round so that the available goods can burn off as quickly as possible in order to achieve $y_3 \leq x$. In this way, he can charge 3 from period 2, which will be accepted by the high valuation consumer.

In a sharp contrast to the classic problem, making an unacceptable offer can be a part of an optimal pricing sequence. By reducing the available amount of goods, the monopolist can credibly refuse to make any sales. As a result, the dynamic market is exposed to two different sources of inefficiency. First, as in the classic problem, it will take more than one period to serve the consumers and this delay will be greater than in the classic problem if the monopolist chooses to burn off some of the goods. Second, if the monopolist opts for letting the goods get burned off, then some consumers may not be served. One of our objectives is to understand how the perishability factor affects the overall inefficiency of the market outcome, especially when $\Delta \rightarrow 0$.

A natural state variable is (x, y) which characterize the residual demand and the available goods. If $x \geq \beta^{-1}y$, then the monopolist can charge 3 and serves all remaining high valuation consumers. The key decision is how long the monopolist has to wait before he can credibly charge 3.

Essentially, the monopolist has three options at (x, y) .

- **Final sale.** Charge 1 to serve everyone in the market. His profit will be

$$(3.6) \quad y.$$

- **Accelerating.** Charge $3 - 2\delta$ which is accepted by all high valuation buyers, and in the following round, charge 1 which is accepted by the remaining low valuation buyers. The average discounted profit is

$$(3.7) \quad (3 - 2\delta)x + \delta\beta(y - x).$$

- **Delaying.** Continue to charge 3 until the high valuation consumer concludes that the monopolist will not lower the price, which will make the high valuation buyer to accept the offer. Let k be the first period that

$$\beta^{k-1}y \leq x$$

when the monopolist can charge credibly 3, which is accepted by the high valuation buyer immediately. Thus, if it takes k rounds, the expected profit is

$$(3.8) \quad 3\delta^{k-1} \min(\beta^{k-1}y, x).$$

In order to delineate the optimal action of the monopolist under (x, y) , let us characterize the “indifference state” between the Coase conjecture type strategy and the last one. That is state (x, y) solving

$$(3.9) \quad \max(y, (3 - 2\delta)x + \delta\beta(y - x)) = 3\delta^{k-1} \min(\beta^{k-1}y, x).$$

assuming for a moment that k can take any positive real number.

Lemma 3.1. (1) $\forall(x, y), \exists k \geq 0$ such that (3.9) holds.

(2) If (x, y) satisfies (3.9), then so does $(\lambda x, \lambda y) \forall \lambda > 0$.

(3) Define

$$\mathcal{K} = \{k : \exists(x, y) \text{ such that (3.9) holds.}\}.$$

Then, \mathcal{K} is a compact and connected set and therefore,

$$\sup \mathcal{K} < \infty.$$

(4) For a fixed x , and $y' > y$. Let k and k' be the solution associated with (x, y) and (x, y') in (3.9). Then, $k' < k$.

Proof. Define

$$g(k) = \max(y, (3 - 2\delta)x + \delta\beta(y - x)) - 3\delta^{k-1} \max(\beta^{k-1}y, x).$$

which is a continuous function of k . Note $g(0) < 0$ and $\lim_{k \rightarrow \infty} g(k) > 0$. Moreover, $g(k)$ is a strictly increasing function of k . Thus, there exists a unique k satisfying (3.9). Note that $g(k)$ is a linear function of (x, y) which implies the second statement. We know that the mapping $(x, y) \mapsto k$ satisfying (3.9) is continuous. Since (x, y) is contained in a compact set, \mathcal{K} is compact, which implies the third statement. The last statement can be proved through taking a total derivative of (3.9). \square

For a fixed x , there is one-to-one correspondence between (x, y) and the solution k from (3.9). For each k , define $\alpha(k) = y/x$ where (x, y) induces k as the solution of (3.9). From Lemma 3.1,

$$\mathcal{U}(k) = \{(x, y) : k \text{ is the solution of (3.9)}\}$$

is a half line through the origin with slope $\alpha(k)$ which is a strictly decreasing function of k . The slope of $\alpha(k)$ can range from 1 to $+\infty$. Moreover, as $\Delta \rightarrow 0$ (so that $\delta, \beta \uparrow 1$), $\sup \mathcal{K} \uparrow \infty$.

$\mathcal{U}(k)$ represents the collection of states that make the monopolist indifferent between two options of accelerating and delaying *if the delay takes k periods*. However, there is no guarantee that k is self-fulfilled. Again, let us assume for another moment that k can take any positive real number. Given (x, y) , we can find a unique $k > 0$ such that

$$\beta^{k-1}y = x$$

which is the first time when the monopolist can credibly charge 3, which is accepted by the remaining high valuation buyer with probability 1. Define

$$\mathcal{V}(k) = \left\{ (x, y) : y = \frac{1}{\beta^{k-1}}x \right\}$$

as the collection of states, which takes k periods to reach the area

$$\{(x, y) : y \leq x\}$$

where the monopolist's offer 3 is accepted with probability 1. Note that $\mathcal{V}(k)$ is a half-line passing through the origin. Its slope is ranging from 1 to $+\infty$, which is a strictly increasing function of k .

Therefore, there exists $k^* > 0$ such that

$$(3.10) \quad \mathcal{V}(k^*) = \mathcal{U}(k^*).$$

This k^* has a special meaning in the sense that if $(x, y) \in \mathcal{U}(k^*)$, the monopolist expects that in k^* periods, his offer 3 will be accepted with probability 1 and indeed, it takes k^* period before such event occurs.

Remark 3.2. *If k can take only a positive integer value, the same analysis proves the existence of a positive integer k^* such that*

$$(3.11) \quad \frac{1}{\beta^{k^*}} \geq \alpha(k^*) \geq \frac{1}{\beta^{k^*-1}}.$$

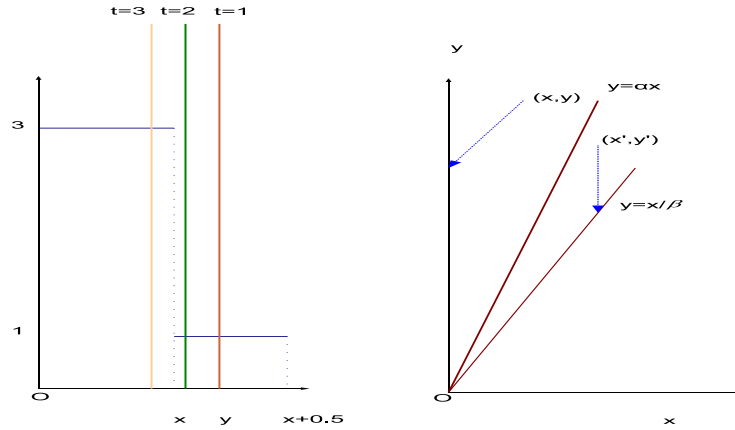


Figure 1: The left panel illustrates how the available stock decays in case that the monopolist makes no sales in the first two rounds. The right panel depicts the area of (x, y) associated with the two different strategies. The bold straight line is $y = \alpha x$. The monopolist makes an acceptable offer immediately if $y > \alpha x$. Note that if the monopoly makes sales, x and y decreases by the same amount, and (x, y) is moving along the 45 degree line passing through (x, y) . If all high valuation consumer is served, then x becomes 0. If $y' < \alpha x'$, then the state moved down vertically because no sales are made until the state hits $y = x/\beta$.

According to the definition of $\alpha(k^*)$ and $\mathcal{U}^*(k^*)$, if $y > \alpha(k^*)x$, then the monopolist charge 1 or $3 - 2\delta$ to 3, depending upon the size of x . Unless $x > 0$ is too small, the monopolist immediately makes an offer $3 - 2\delta$ which is accepted by all high valuation seller whose mass is x , and offer 1 to clear the market. As depicted in Figure 1, the state moves along 45 degree line passing through (x, y) , because each consumer demand exactly one unit. On the other hand, if $x/\beta < y < \alpha(k^*)x$, then the monopolist refuses to make an acceptable offer. For analytic convenience, let us assume that the monopolist charge $3 + \epsilon$ for a small $\epsilon > 0$, which is rejected by all high valuation consumer. The available stock decays at the rate of β in each period. After k periods of rejected offers, suppose

that $\beta^k y < x < \beta^{k-1} y$ holds. If the high valuation consumer rejects $3 + \epsilon$, then $\beta^k y < x$ implies that from the next period, there is excess demand among high valuation consumer and the monopolist can charge 3. Thus, all high valuation consumer is willing to accept any offer up to 3. Knowing this, the monopolist charge 3, following k unacceptable offers.

We can sustain this outcome as the subgame perfect equilibrium.

Proposition 3.3. *The above outcome path can be sustained by a subgame perfect equilibrium, which involves randomization off the equilibrium path.*

Proof. See Appendix B. □

3.2. Properties. The equilibrium strategy may entail a positive amount of time when the monopolist is willing to make no acceptable offers. Wasting time without making sales can never be a part of an equilibrium strategy in a classic durable good problem if the gain from trading is common knowledge, as in our example. However, because the goods are perishable, however slightly, making no sales does not mean wasting time. Rather, the monopolist can deliberately wasting some available stocks in order to manipulate the beliefs of the consumers about the monopolist's future prices.

Some consumers whose valuations are higher than the production cost may not be served. In a static monopoly problem, if the demand curve is inelastic, the monopolist finds it profitable to reduce the total sales. The same intuition applies here. Because the monopolist may decide to waste some existing stock, which requires time, it takes more time to complete the sales. As a result, even if almost all consumers are served, one cannot conclude that the market outcome is almost efficient. If it takes substantial time to achieve an optimal amount of stocks, the realization of the gains from trading can take excessively long time. As a result, the discounted social surplus could be very small, even if almost all consumers eventually purchase the goods.

While this example is simple enough to allow us to calculate the subgame perfect equilibrium, it has a couple of rather peculiar features. Because the type space of the consumers is discrete, the gain from the reducing the available stock increase discontinuously. Combined with the fact that the initial stock is smaller than the whole market demand ($y \leq x + 0.5$), the monopolist has good reason to delay the offer, because a large return from the delay is realized fairly quickly. A natural question is whether the key properties of the equilibrium are carried over to the cases where the demand curve is continuous and the initial stock is larger than the whole market demand. To answer this question, we examine the market with a linear demand for the rest of the paper.

4. MARKET WITH A LINEAR DEMAND

In order to highlight the key features of the equilibrium which will be constructed in this section, let us first examine a simple, but artificial, game. Then, we construct a reservation price equilibrium, which is approximated by the equilibrium of the artificial game.

4.1. Example 2. An Artificial Game. Let us consider an artificial game in which the monopolist in the market with a linear demand curve (2.1) has two options: make one final sale, or delay the sale. The monopolist can choose when he opens the market, say

$\tau_f \geq 0$, and then, he must make an offer to serve everyone in the market or sell all the goods available at that point, if there is an excess demand.

If the monopolist charges $p_{\tau_f} = 1 - q_{\tau_f}$, q_{τ_f} portion of consumers will be served. In any equilibrium, q_{τ_f} is selected in such a way that the monopolist cannot improve his profit by delaying the sale. Define

$$h(q : \tau) = e^{-r\tau} \left[e^{-\tau b} q (1 - e^{-\tau b} q) \right] - q(1 - q)$$

as the gain from delaying τ amount of real time and charging $1 - e^{-\tau b} q$ to serve everyone whose valuation is higher than $1 - q$ in the market, if the present available stock is $q \leq 1$.

It is easy to see that $h(q : 0) = 0$ and

$$\frac{\partial h(q : 0)}{\partial \tau} = -(r + b)q + (r + 2b)q^2.$$

If

$$\frac{\partial h(q : 0)}{\partial \tau} \leq 0,$$

then $\forall \tau > 0$, $\frac{\partial h(q : \tau)}{\partial \tau} < 0$. If

$$\frac{\partial h(q : 0)}{\partial \tau} \geq 0,$$

then $\forall q' \geq q$, $\frac{\partial h(q' : 0)}{\partial \tau} \geq 0$. If $\partial h(q : 0) / \partial \tau > 0$, then the monopolist can be better off by delaying the sale on period. Although the total stock will be reduced to $e^{-\Delta b} q$, but he can credibly charge higher price $1 - e^{-\Delta b} q$ to generate higher profit. Similarly, if $\partial h(q : 0) / \partial \tau < 0$, then he should have accelerated the sale. Thus, the optimal quantity q solves

$$\frac{\partial h(q : 0)}{\partial \tau} = 0,$$

which is

$$(4.12) \quad q = \frac{r + b}{r + 2b},$$

and the discounted profit is

$$e^{-r\tau(y)} \frac{b(r + b)}{(r + 2b)^2}$$

where

$$e^{-b\tau(y)} y = \frac{r + b}{r + 2b}.$$

The optimal choice of the quantity of the monopolist $(r + b) / (r + 2b)$ is very intuitive. If the monopolist is very impatient so that $r > 0$ is very large, the monopolist is not willing to delay and as a result, the total amount of goods delivered is close to 1. On the other hand, if the good is perishing quickly so that $b > 0$ is large, the quantity converges to $1/2$ which is the monopolistic profit maximizing quantity. The monopolist can use the rapid decay to exercise his monopolistic power.

The ensuing analysis shows that the outcome of this artificial game approximate the outcome of the reservation price equilibrium of the dynamic monopoly problem where he can charge a series of prices over time, combined with delaying the offers. The missing step is to make it sure that the delay strategy generates a higher profit than the strategy

satisfying the Coase conjecture, from which the monopolist can generate profit $q_f(1 - q_f)$ almost instantaneously if $\Delta > 0$ is small. Note that

$$e^{-r\tau(y)} \frac{b(r+b)}{(r+2b)^2} > q_f(1 - q_f)$$

holds as long as q_f is sufficiently close to 1 for given b, r, y .³ Then, a substantial delay of an acceptable offer can arise in a reservation price equilibrium.

Note that for a fixed $r > 0$,

$$\lim_{b \rightarrow 0} \frac{r+b}{r+2b} = 1$$

which implies that every consumer will be served in the limit. Yet, the outcome is extremely inefficient. To see this, define

$$\tau_f(b, r) = \lim_{\Delta \rightarrow 0} \Delta \tau_f(\Delta)$$

as the amount of real time to complete the sales in the limit as $\Delta \rightarrow 0$. Once the monopolist begins to make an acceptable offer in a reservation price equilibrium, he completes the sales quickly as $\Delta \rightarrow 0$, as dictated by the Coase conjecture. Thus, $\tau_f(b, r)$ can be approximated by the first time to make an acceptable offer. A simple calculation shows that

$$\tau_f(b, r) = \frac{1}{b} \left(\log y - \log \frac{r+b}{r+2b} \right).$$

If $y > 1$, as $b \rightarrow 0$, the right hand side increases indefinitely, implying that the monopolist is willing to delay the sale as long as possible in order to generate a positive profit, even if it is realized after a long delay. As a result, the market outcome becomes extremely inefficient, because the potential gains from trading is discounted away during the long delay. Even if $y = 1$, l'Hôpital's rule implies that

$$\lim_{b \rightarrow 0} \tau_f(b, r) = \frac{1}{r}$$

which implies that the delay does not vanish and can be significant if the monopolist is very patient.

4.2. Construction of a Reservation Price Equilibrium. We search for a reservation price equilibrium where the equilibrium path consists of two phases: the first phase where the monopolist is making unacceptable offers, and the second phase where the monopolist is making a series of acceptable offers. In the second phase, we can invoke the same insight as in the classic problem to construct the equilibrium path. In particular, we can write (2.2) in a simpler form (2.4). And, then by calculating the optimal time for delaying to make the first acceptable offer, we construct the equilibrium outcome, where the total gains from trading vanishes as $\Delta \rightarrow 0$.

To write the formal statement, let $\mathcal{W}_c(\Delta)$ and $\mathcal{W}_s(\Delta)$ be the consumer and the producer surplus if the time between the offers is $\Delta > 0$.

³One might wonder whether we have to check the same inequality for each $\tau > 0$. From the analysis of $h(q : \tau)$, we know that if this equality holds the beginning of the game, then it continues to hold for $\tau > 0$ until the available stock reaches the optimal level.

Proposition 4.1. $\forall \epsilon > 0, \forall r > 0, \exists \bar{b} > 0, \forall b \in (0, \bar{b}], \exists \bar{q}_f$ such that $\forall q_f \in [\bar{q}_f, 1), \exists \bar{\Delta} > 0$ such that $\forall \Delta \in (0, \bar{\Delta}), \mathcal{W}_c(\Delta) < \epsilon$ and $\mathcal{W}_s(\Delta) < \epsilon$.

We construct an equilibrium for the rest of the section. As in Section 4.1, let us assume for a moment that the monopolist can choose any real time $\tau_1 \geq 0$ to start to make an acceptable offer, in order to avoid the integer problem. Given residual demand $D(0, q_f)$ and the initial stock y with $q_f < 1$, the optimization problem can be written as

$$(4.13) \quad \max_{\tau_1 \geq 0, \mathbf{q} \in \mathbf{Q}} e^{-r\tau_1} \sum_{t=1}^{\infty} p_t(q_t - q_{t-1})\delta^{t-1}$$

$$(4.14) \quad \text{subject to} \quad (1 - q_t) - p_t = \delta((1 - q_t) - p_{t+1})$$

$$(4.15) \quad p_{T_f} = 1 - q_{T_f}$$

$$(4.16) \quad \beta^{T_f} \left(e^{r\tau_1} y - \sum_{t=1}^{T_f} \beta^{-t}(q_t - q_{t-1}) \right) \geq 0$$

$$(4.17) \quad \beta^{T_f} \left(e^{r\tau_1} y - \sum_{t=1}^{T_f} \beta^{-t}(q_t - q_{t-1}) \right) (q_{T_f} - q_f) = 0$$

where T_f is the number of periods when a positive portion of consumers is served. τ_1 is the time during which the monopolist makes no sale, simply burning off the available stock at the rate of e^{-b} . The objective function and the first two constraints are identical to the classic problem and so is the definition of T_f .

The last two constraints warrant some explanation, as they capture the key elements of the perishable problem. The first step is to observe that the trading must be completed in finite rounds.

Lemma 4.2. *If $q_f < 1$, then in any optimal solution, $\tau_1 + \Delta T_f < \infty$.*

Proof. Given the structure of the candidate equilibrium, the proof to show $T_f < \infty$ is identical with the one in the classic problem [9, 10]. It remains to show that $\tau_1 < \infty$.

It suffices to show that $\exists \tau^* > 0$ such that if $\tau_1 > \tau^*$, then (τ_1, \mathbf{q}) cannot be an optimal solution for any $\mathbf{q} \in \mathbf{Q}$.

Given demand curve $D(0, q_f)$, let $q^m(0, q_f)$ be the static monopoly profit maximizing quantity. The monopolist can choose τ_1 so that $e^{-b\tau^*} y_0 = q^m(0, q_f)$, and charge $1 - q^m(0, q_f)$, which will be accepted by all consumers whose valuation is at least $1 - q^m(0, q_f)$. Thus, the equilibrium payoff of the monopolist is uniformly bounded from below by

$$e^{-b\tau^*} (1 - q^m(0, q_f)) q^m(0, q_f).$$

If the monopolist spends more than τ^* before making an acceptable offer, he cannot achieve this level of profit. Thus, if τ_1 is selected in an equilibrium, then $\tau_1 \leq \tau^*$. \square

If the monopolist sells $q_1 - q_0$ in the initial round, then at the end of the period, $y - (q_1 - q_0)$ is available, but by the beginning of period 2, only $\beta(y - (q_1 - q_0))$ is available. Thus, by the time when all available goods are sold,

$$\beta(\cdots(\beta(y - (q_1 - q_0)) - (q_2 - q_1))) - (q_{T_f} - q_{T_f-1}) \geq 0$$

must hold, because the amount of sales in period t cannot exceed the amount of stocks available in that period. The constraint can be written as

$$\beta^{T_f} \left(y - \sum_{t=1}^{T_f} \beta^{-t} (q_t - q_{t-1}) \right) \geq 0.$$

However, if $q_{T_f} = q_f$, then it is possible that a positive amount of goods is left over. This happens when no remaining consumer wants the good. but, if $q_{T_f} < q_f$, the final offer must be such that all the remaining consumer must be served, and therefore, (4.16) must hold with equality. Hence, the complementary slackness condition (4.17) must hold.

We show by construction that the above optimization problem has a solution.

Proposition 4.3. *Suppose that the initial stock $y > 1$ and the size of the demand $q_f < 1$. Given demand curve $(0, q_f)$ and initial stock y , there exists $\bar{\Delta} > 0$ such that $\forall \Delta \in (0, \bar{\Delta})$, there exists an optimal solution (τ_1, \mathbf{q}) .*

Proof. See Appendix C. □

Now, let us consider the original problem where the monopolist can make a decision, including the one for delay as well as for the offer, can be made every $\Delta > 0$ units of time. Let T_1 be the number of periods between the beginning of the game and the first time when the monopolist makes an acceptable offer. Then, $\tau_1 = \Delta T_1$. Given $\tau_1 > 0$ selected, we can choose T_1 so that the profit of the monopolist is maximized among $T_1 \in \{1, \dots, \lceil \tau^*/\Delta \rceil\}$.

Theorem 4.4. *Given $q_f < 1$ and $y > 1$, $\exists \bar{\Delta} > 0$ such that $\forall \Delta \in (0, \bar{\Delta})$, $\exists T_1 \in \{0, 1, 2, \dots\}$ and $\mathbf{q} = (q_1, q_2, \dots)$ that solves (4.13) where $\tau_1 = \Delta T_1$ can take only non-negative integer values.*

The next step is to show that the constructed outcome path can be sustained as a reservation price equilibrium.

Proposition 4.5. *The optimal outcome path in Theorem 4.4 can be sustained by a reservation price equilibrium.*

Proof. See Appendix D □

By letting $q_f \rightarrow 1$, we have a reservation price equilibrium for $D(0, 1)$, which involves the delay of the first acceptable offer, while burning off the existing stock. It would be interest to see how the inefficiency changes as $\Delta \rightarrow 0$.

Fix y , and let $q_{T_f(\Delta)}$ be the total amount that is delivered and $T_1(\Delta)$ be the first round when the monopolist is making an acceptable offer when the time between the offers is $\Delta > 0$. Clearly, $\forall \Delta > 0$, $q_{T_f(\Delta)} \in [0, y]$ and $\Delta T_1(\Delta) \in [0, \tau^*]$. Define

$$q_{T_f(0)} = \lim_{\Delta \rightarrow 0} q_{T_f(\Delta)}$$

and

$$\tau_1(0) = \lim_{\Delta \rightarrow 0} \Delta T_1$$

by taking a convergent subsequence, if necessary.

Note that the sequence of acceptable offers is precisely the same as the one in the classic counter part where the demand curve is $D(0, q_{T_f})$. Hence, the Coase conjecture implies that the profit from the perishable problem converges to

$$e^{-\tau_1 b} q_{T_f(0)} (1 - q_{T_f(0)}).$$

Hence, the limit properties of the reservation price equilibrium can be examined through the same method as illustrated in Example 4.1.

Let $\mathcal{W}_c(\Delta)$ and $\mathcal{W}_s(\Delta)$ be the expected consumer surplus and the expected producer surplus from the game where the time between the offers is $\Delta > 0$. The following proposition formalizes this observation.

Proposition 4.6. $\forall \epsilon > 0, \forall b > 0 \exists \bar{q}_f$ such that $\forall q_f \in (\bar{q}_f, 1), \exists \bar{\Delta} > 0$ such that $\forall \Delta \in (0, \bar{\Delta}), \mathcal{W}_c(\Delta) < \epsilon$ and $\mathcal{W}_s(\Delta) < \epsilon$.

5. SMALL COMMITMENT BUT LARGE PROFIT

Proposition 4.6 confirms our intuition that the monopolist's profit largely depends upon his ability to make commitment. If $\Delta > 0$ and $b > 0$ are small, the monopolist has little ability to commit not to sell. Thus, the profit should be low.

While this argument sounds very plausible, the underlying reasoning misses an important strategic opportunity of the monopolist. If the goods slowly perish (small $b > 0$), the monopolist can credibly delay to make an acceptable offer for a long time. If the consumer knows that the lower price will not be offered any time soon, he is willing to accept a higher price than otherwise. Exploiting the impatience of the consumer, the monopolist can generate large profit.

This section makes this observation precise. As in Section 4, we start with a simple artificial game to explore the key properties of the equilibrium we shall construct. Then, we construct a reservation price equilibrium, which generates expected profit close to the static monopoly profit for a small $b > 0$.

5.1. Another Artificial Game. The monopolist uses the delay tactic as a way to influence the consumer's belief about the future prices offered by the monopolist. Yet, the delay tactic has an obvious downside, especially if the beginning of the game is delayed as in the equilibrium constructed in Section 4.2: the monopolist has to delay the realization of the profit. Because the consumers with high reservation value is willing to pay higher price, the monopolist has to balance the benefit of delaying and burning the available stock against the cost of delaying the profit, especially against the high reservation value consumers.

To explore the tension between these two strategic motivations, let us examine a slightly more elaborate version of Example 4.1 where the monopolist can only delay the beginning of the game. Instead, let us allow the monopolist to choose a time interval with length $\tau > 0$ during which he chooses to burn the stock at the instant rate of e^{-b} . Thus, the sales can occur twice, before and after the τ break. Let (q_1, q_2) represent the total amount of goods delivered after each sales, and (p_1, p_2) be the respective delivery prices. That is, at the beginning of the game, the monopolist charges p_1 to serve q_1 , and then, takes break for τ time. After the break, he charges p_2 to serve additional $q_2 - q_1$ consumers. As in

Example 4.1, the initial quantity of the goods is y . All other parameters of the models remain the same as in Example 4.1.

We calculate the optimal solution through backward induction. Suppose that q_1 has been served. Then, $y - q_1$ is available, and the residual demand curve is $D(q_1, q_f)$. Throughout this example, we choose both $y > 1$ and $q_f < 1$ sufficiently close to 1, and $b > 0$ sufficiently small. Invoking the same logic as we did in Section 4.1., we have

$$q_2 - q_1 = (1 - q_1) \frac{r + b}{r + 2b}$$

and the monopolist has to delay the offer p_2 by τ in order to satisfy the market clearing condition:

$$(5.18) \quad e^{-b\tau}(y - q_1) = q_2 - q_1 = (1 - q_1) \frac{r + b}{r + 2b}$$

which implies that

$$p_2 = (1 - q_1) \frac{b}{r + 2b}.$$

Let $\tau(q_1)$ be the solution for (5.18). Note that $\tau'(q_1) > 0$. In order to make consumer q_1 indifferent between p_1 and p_2 ,

$$(1 - q_1) - p_1 = e^{-r\tau(q_1)} \left((1 - q_1) - (1 - q_1) \frac{b}{r + 2b} \right)$$

which implies that

$$p_1 = (1 - q_1) \left[1 - e^{-r\tau(q_1)} \frac{r + b}{r + 2b} \right].$$

Hence, the profit from selling q_1 in the first round can be written as

$$\begin{aligned} V(q_1) &= q_1(1 - q_1) \left[1 - e^{-r\tau(q_1)} \frac{r + b}{r + 2b} \right] + e^{-r\tau(q_1)}(1 - q_1)^2 \frac{(r + b)b}{(r + 2b)^2} \\ &= (1 - q_1) \left(e^{-r\tau(q_1)} \frac{(r + b)b}{(r + 2b)^2} + q_1 \left[1 - e^{-r\tau(q_1)} \frac{(r + b)(r + 3b)}{(r + 2b)^2} \right] \right). \end{aligned}$$

Note that as $b \rightarrow 0$, $V(q_1)$ converges uniformly to

$$(1 - e^{-r\tau(q_1)})(1 - q_1)q_1$$

over $q_1 \in [0, q_f]$. As $\tau(q_1)$ is determined by (5.18), the first order condition is

$$V'(q_1) = (1 - e^{-r\tau(q_1)})(1 - 2q_1) + \frac{rq_1(1 - q_1)(y - 1)}{b(y - q_1)^2} = 0.$$

Note that as $y \downarrow 1$, the second term in the first order condition vanishes, and the optimal value q_1 converges to 0.5. In addition, (5.18) implies that $\tau(q_1) \rightarrow \infty$ as $b \rightarrow 0$, as long as $y > 1$. Thus, the delivery price of q_1 converges to $p_1 = 0.5$ and the profit converges to the static monopolist's profit. The slow rate of decay combined with a negligible profit from the continuation game renders the long delay of an offer a credible strategy of the monopolist.

5.2. Another Reservation Price Equilibrium. The key feature of the equilibrium constructed in Section 4.2 is that the monopolist can credibly delay to make an acceptable offer, especially when the expected profit from accelerating the sale is small. Following the same logic as in Section 4.2, we can construct another reservation price equilibrium, which generates the monopolist the static monopolist's profit as $\Delta \rightarrow 0$ as illustrated in Section 5.1. Because the basic idea is identical, let us only sketch the construction process for demand curve $D(0, q_f)$ with $q_f < 1$ and $y > 1$.

Proposition 5.1. $\forall \epsilon > 0, \exists \bar{b} > 0, \exists \bar{y} > 1, \forall b \in (0, \bar{b}), \forall y \in (1, \bar{y}), \exists \bar{q}_f, \forall q_f \in [\bar{q}_f, 1), \exists \bar{\Delta} > 0, \forall \Delta \in (0, \bar{\Delta}), \mathcal{W}_s^m \leq \mathcal{W}_s(\Delta) + \epsilon$ where \mathcal{W}_s^m is the static monopolist profit.

Let us consider a residual demand $D(q_0, q_f)$ with $q_f < 1$ which will be later increased toward 1, and imagine an equilibrium that consists of two phases. In each phase, the monopolist is making a series of acceptable offers, denoted as $\{p_{1,t}\}_{t=1}^{T_{1,f}}$ and $\{p_{2,t}\}_{t=T_{q,f}+T_0+1}^{T_f}$ where the subscript represents the phase and the period. T_0 represents the number of periods during which the monopolist is making unacceptable offers. Fix \bar{q}_1 , which is the end of the first phase, which will be served by the time when the monopolist offers $T_{1,f}$.

By invoking the same logic as Lemma 2.3, we can show that $T_f < \infty$ and $\limsup_{\Delta \rightarrow 0} \Delta T_f < \infty$. Let us write down the optimization problem of the monopolist for a given \bar{q}_1 .

$$(5.19) \quad \max_{T_0 \geq 0, \mathbf{q} \in \mathbf{Q}} \sum_{t=1}^{T_{1,f}} (q_t - q_{t-1}) \delta^{t-1} + \delta^{T_0} \sum_{t=1}^{\infty} p_t (q_t - q_{t-1}) \delta^{t-1}$$

$$(1 - q_t) - p_t = \delta((1 - q_t) - p_{t+1}) \quad \forall t \neq T_{1,f}$$

$$(5.20) \quad \text{s.t.} \quad (1 - q_{T_{1,f}}) - p_{T_{1,f}} = \delta^{T_0}((1 - q_{T_{1,f}}) - p_{T_{1,f}+T_0+1})$$

$$(5.21) \quad q_{T_{1,f}} = \dots = q_{T_{1,f}+T_0}$$

$$(5.22) \quad p_{T_f} = 1 - q_{T_f}$$

$$(5.23) \quad \beta^{T_{1,f}} \left(y - \sum_{t=1}^{T_{1,f}} \beta^{-t} (q_t - q_{t-1}) \right) = y - \bar{q}_1$$

$$(5.24) \quad \beta^{T_f - T_{1,f} - T_0} \left(\beta^{T_0} (y - \bar{q}_1) - \sum_{t=1}^{T_f - T_{1,f} - T_0} \beta^{-t} (q_t - q_{t-1}) \right) \geq 0$$

$$(5.25) \quad \beta^{T_f - T_{1,f} - T_0} \left(\beta^{T_0} (y - \bar{q}_1) - \sum_{t=1}^{T_f - T_{1,f} - T_0} \beta^{-t} (q_t - q_{t-1}) \right) (q_{T_f} - q_f) = 0$$

The optimization problem is virtually identical with (4.13), except that (4.16) is now split into two parts: (5.23) before and (5.24) after T_0 rounds of unacceptable offers.

After \bar{q}_1 consumers are served, the continuation game is played according the equilibrium strategy constructed in Section 4.2 associated with residual demand $D(\bar{q}_1, q_f)$. Then, (5.20) ensures that consumer \bar{q} is indifferent between the last offer $p_{T_{1,f}}$ before the break and $p_{T_{1,f}+T_0+1}$ after the break. Given $p_{T_{1,f}}$, we construct $p_1, \dots, p_{T_{1,f}}$ as in the classic durable goods monopoly problem with $p_{T_{1,f}}$ being the final offer.

Let $V(q_0, q_f, \bar{q}_1)$ be the optimal value of (5.19). Define

$$V(q_0, q_f) = \max_{\bar{q}_1 \in [q_0, q_f]} V(q_0, q_f, \bar{q}_1)$$

as the value function obtained by choosing \bar{q}_1 optimally, and $\mathbf{q}(q_0, q_f)$ be the associated optimal outcome path.

By setting $q_0 = 0$ and letting $q_f \rightarrow 1$, we have the outcome path for demand curve $D(0, 1)$. One can prove that the outcome path can be sustained as a reservation price equilibrium path, following the same reasoning as Proposition 4.5. By the construction, the series of acceptable offers satisfy the Coase conjecture. More precisely,

$$\lim_{\Delta \rightarrow 0} p_1 - p_{T_{1,f}} = 0$$

and

$$\lim_{\Delta \rightarrow 0} p_{T_{1,f}+T_0+1} - p_{T_f} = 0.$$

Hence, the limit properties of the reservation price equilibrium can be approximated by the equilibrium illustrated in Section 5.1.

6. CONCLUDING REMARKS

6.1. Delayed offer. In order to highlight the impact of the perishability to the Coase conjecture, we literally follow the rule of the classic durable goods monopoly problem, forcing the monopoly to announce a high price in order to delay the game. Thus, the delay occurs as a positive integer multiple of $\Delta > 0$.

A more general, perhaps more natural, formulation would be to let the monopolist to delay the bargaining continuously. That is, following each history, the monopolist can choose a pair of numbers, (p, τ) : p is offered in τ time. Given p , consumers decided to accept or reject. If the offer is rejected, then the monopolist has to wait $\Delta > 0$ unit of time before making another move.

In the classic problem, the monopolist has no reason to delay: $\tau = 0$ following every history.⁴ Thus, the Coase conjecture holds. Because the monopolist can delay the bargaining continuously, the analysis is in fact simpler and closer to the examples where we assume that the game is delayed continuously.

6.2. Folk Theorem. The equilibrium constructed in Section 4.2 can serve as a credible threat to force the monopolist to follow a designated outcome path. Following the same idea as in [2], we can obtain the folk theorem if $\Delta \rightarrow 0$ and then $b \rightarrow 0$. The equilibrium constructed in Section 5.2 is remarkable in the sense that the restriction imposed off the equilibrium path by the reservation price equilibrium does not force the equilibrium payoff of the monopolist to be small, as in the classic durable goods problem.

⁴This is true because the monopolist has no private information. If the monopolist has private information, then the analysis of [1] implies that the monopolist may have incentive to delay.

6.3. General Demand Curves. In order to construct a reservation price equilibrium, we use the property that $T_f(q_o, q_f, y)$ is an increasing function of q_f . This is crucial in constructing a strategy off the equilibrium path. However, this monotonic property of T_f does not hold in general. In particular, when there is a large mass assigned to the lower reservation value consumers, the increase of q_f may prompt the monopolist to accelerate the sales in order to meet the large demand of the low reservation value consumers. In this case, the reservation price equilibrium may fail to exist. Yet, we can construct a subgame perfect equilibrium following the same idea, although the lower price from the monopolist may be accepted by fewer consumers, who expect a lower price sooner than along the equilibrium path.

6.4. Regulation of a Durable Good Monopolist. The Coase conjecture has a profound implication to the regulatory policy of the monopolist, as it reveals the source of the monopolistic power. A natural remedy is to weaken the commitment capability of the monopolist, for example, by forcing the sales instead of the lease. However, the discontinuity with respect to the commitment power raises a challenge to the conventional wisdom. Virtually all durable goods are subject to slow decay because of physical deterioration or technological obsolescence. Given that a slight decay can generate a substantial monopolistic power, we need to search for a new idea of regulating a monopolist other than weakening the commitment power.

APPENDIX A. PROOF OF LEMMA 2.3

Because all results except for the last one are known from the existing literature, we only prove the last statement. We use the linearity of the demand curve to prove that $T_f(q_0, q_f, y)$ is an increasing function of q_f . We use the linearity again to prove the last statement. Without loss of generality, we focus on the case where $q_0 = 0$.

Consider the classic problem parametrized by the demand curve $D(0, q_f)$. Here, the consumer with the lowest valuation is $1 - q_f$. Suppose that the lowest valuation of the consumer changes from $1 - q_f$ to $\alpha(1 - q_f)$ where $\alpha \in (0, 1)$. This change is equivalent to increase q_f to $1 - \alpha + \alpha q_f$. The monopolist faces essentially the identical optimization problem if the demand curve is $D((1 - \alpha), (1 - \alpha) + \alpha q_f)$.

This follows from the fact that $D(1 - \alpha, (1 - \alpha) + \alpha q_f)$ is obtained by a positive affine transformation of the original linear demand $D(0, q_f)$. If $\mathbf{q}(0, q_f, y)$ is an optimal solution associated with demand curve $D(0, q_f)$, then $\alpha \mathbf{q}(0, q_f, y)$ is an optimal solution for $D(1 - \alpha, (1 - \alpha) + \alpha q_f)$. Therefore, it takes exactly the same number of periods to complete the trading under the two different demand curves: $T_f(0, q_f, y_0) = T_f(1 - \alpha, (1 - \alpha) + \alpha q_f, y)$.

Now, consider demand curve $D(0, (1 - \alpha) + \alpha q_f)$. Note that if the trading is completed faster, then the amount of available stock to prompt the seller to initiate the acceptable offer decreases. Once $1 - \alpha$ portion of the consumers is served, it will take $T_f(1 - \alpha, (1 - \alpha) + \alpha q_f, y) = T_f(0, q_f, y_0)$ periods to complete the trading in any optimal path. Thus, in any optimal path, it takes at least $T_f(0, q_f, y_0) + 1$ periods to complete the trading. In particular, if the seller serves initial $1 - \alpha$ consumers and then then follow the optimal pricing path, then such a path will require the least initial stock to prompt the acceptable offers. Hence, if $\mathbf{q}(1 - \alpha, (1 - \alpha) + \alpha q_f, y) = \{q_t\}$ is an optimal path associated with demand curve $D(1 - \alpha, (1 - \alpha) + \alpha q_f, y)$, then

$$\begin{aligned} y_f(1 - \alpha, (1 - \alpha) + \alpha q_f, y) &\geq \frac{(1 - \alpha) + \sum_{t=1}^{T_f(1 - \alpha, (1 - \alpha) + \alpha q_f, y)} (q_t - q_{t-1}) \beta^{-t}}{\beta} \\ &= \frac{(1 - \alpha) + \alpha y_f(1 - \alpha, (1 - \alpha) + \alpha q_f, y)}{\beta} > y_f(1 - \alpha, (1 - \alpha) + \alpha q_f, y) \end{aligned}$$

if $\alpha \in (\beta, 1)$.

Hence, for any β and for any q_f , if we increase q_f by a sufficiently small amount, then y_f strictly increases. Since this is true for any q_f , we conclude that y_f is a strictly increasing function of q_f .

If the demand curve is linear, then we know that the optimal solution is unique. Combining the upper hemi-continuity of the optimal solution with the strictly increasing property of y_f with respect to q_f , we conclude that $y_f(q_0, q_f, y)$ is a strictly increasing continuous function of q_f .

APPENDIX B. CONSTRUCTION OF STRATEGIES OFF THE EQUILIBRIUM PATH

Recall that because each consumer purchases a single unit, the state moves along the 45 degree line passing through the given state (x, y) , if some consumers purchase the good and $\alpha(k^*) > 1$.

If the initial state $(0.5, y)$ is above $\mathcal{U}(k^*)$, then the construction of the actions off the equilibrium path follows the same idea as the weak stationary equilibrium in [10] with minor twist. We only describe the case where the monopolist charges $3 - 2\delta$ along the equilibrium path. Let \mathcal{U}^* be the half line passing through the origin along which the monopolist is indifferent between charging 1 and $3 - 2\delta$. Let α^* be the slope of \mathcal{U}^* . One can easily show that

$$\alpha^* > \alpha(k^*) > 1.$$

If the initial equilibrium offer is $3 - 2\delta$, then the initial state is located between \mathcal{U}^* and $\mathcal{U}(k^*)$.

If $p > 3 - 2\delta^2$, then no consumer accepts the offer, expecting that in the following period, the monopolist will charge $3 - 2\delta$. If $p < 3 - 2\delta$, then every consumer purchases the good. The state moves from $(0.5, y)$ to $(0, y - 0.5)$, which implies that the monopolist has some goods for future sale, because $y > 0.5$. In the next round following such p , the monopolist charge 1 to serve all low valuation consumers.

If $3 - 2\delta < p < 3 - 2\delta^2$, we first locate a point along

$$y = \frac{\alpha^*}{\beta} x$$

that intersects with the 45 degree line passing through initial state $(0.5, y)$. Let $(0.5 - x^*, y - x^*)$ be such a point. Such p is accepted by x^* portion of high valuation consumers who expects that in the following period, the monopolist randomizes between 1 and $3 - 2\delta$ with probability λ to 1 so that

$$3 - p = \delta(3 - (\lambda + (1 - \lambda)(3 - 2\delta))).$$

In the following round, $\beta(y - x^*)$ is available and the new state $(0.5 - x^*, \beta(y - x^*))$ is located along \mathcal{U}^* where the monopolist is indeed indifferent between 1 and $3 - 2\delta$.

If state (x, y) is below $\mathcal{U}(k^*)$ but $y < x$, then the high valuation consumer accepts any offer $p < 3$. Finally, suppose that state (x, y) is below $\mathcal{U}(k^*)$ but $y > x$. For simplicity, let us assume that the monopolist is indifferent between charging 3 and $3 - 2\delta$ along $\mathcal{U}(k^*)$. The other case follows from the same logic, where the monopolist is indifferent between charging 3 and 1 along $\mathcal{U}(k^*)$.

The monopolist is charging 3 in the equilibrium. If he charges $p > 3$, it is clearly optimal for the consumer to reject the offer with probability 1. If he charges $p \leq 3 - 2\delta$, then every high value consumer accepts the offer with probability 1, expecting that the monopolist will charge 1 in the following round. Indeed, after serving all high valuation consumers, the monopolist still have $\beta(y - x)$ amount for sale in the next round. He charges 1 to serve some of the low valuation consumers.

Suppose that the monopolist charges $p \in (3 - 2\delta, 3)$. Recall that $\alpha(k^*) > 1$. Find a point along

$$y = \frac{\alpha(k^*)}{\beta}x$$

that intersects with the 45 degree line passing through the given state (x, y) . Let $(x - x', y - y')$ be the intersection. Given p , x' portion of consumers accepts the offer, expecting that the monopolist will randomize between 3 and $3 - 2\delta$ in the following round. Indeed, in the following round, the state is $(x - x', \beta(y - y'))$ which is located on $\mathcal{U}(\alpha^*)$, where the monopolist is indifferent between charging $3 - \delta$ and 3.

This completes the construction of the equilibrium strategy. It remains to verify that this configuration constitutes a perfect equilibrium, except for the part where the monopolist cannot benefit from accelerating the sales. In particular, given the fact that the monopolist has to charge 3 which is not accepted by any buyer for a long time, it is not obvious whether or not a slight price cut can increase the profit of the monopolist.

To complete this part of the proof, let us fix state (x, y) . If $y \leq \frac{x}{\beta}$, then the equilibrium offer 3 is accepted with probability 1. Thus, it is obvious that the monopolist has no incentive to lower his price.

Fix $y \in (\frac{x}{\beta^\ell}, \frac{x}{\beta^{\ell+1}}]$ for some $\ell > 1$ but $\ell \leq k^*$. Conditioned on $p < 3$, q portion of high valuation consumer will accept the offer where

$$q = \frac{x - \beta^k y}{1 - \beta^k}$$

so that $(x - q, \beta(y - q))$ is located along $\mathcal{U}(k^*)$ where the monopolist is indifferent between the two pricing rules: charge 3 or follow the path abiding the Coase conjecture (which in this case is $3 - 2\delta$). By charging $p < 3$ in this round, the monopolist can make at most

$$3 - q + \delta^{k^*}(x - q)$$

while by following the equilibrium strategy the monopolist can make at least

$$3\delta^\ell.$$

It suffices to show that

$$3 - q + \delta^{k^*}(x - q) \geq 3\delta^\ell.$$

After substituting q , one can show that this inequality is equivalent to

$$y \geq \frac{1 - \beta^{k^*}}{\beta^{k^*}(1 - \delta^{k^*})} \frac{1 - \delta^{k^*}}{1 - \beta^{k^*}} - (\delta^\ell - \delta^{k^*}) \Big) x.$$

If $\ell \leq k^*$ and $y \geq x/\beta^\ell$, then

$$\frac{x}{\beta^\ell} \geq \frac{1 - \beta^{k^*}}{\beta^{k^*}(1 - \delta^{k^*})} \left(\frac{1 - \delta^{k^*}}{1 - \beta^{k^*}} - (\delta^\ell - \delta^{k^*}) \right) x$$

Therefore, we conclude that $y \in (\frac{x}{\beta^\ell}, \frac{x}{\beta^{\ell+1}}]$, then the monopolist's profit from deviation cannot be larger than the equilibrium payoff.

APPENDIX C. PROOF OF PROPOSITION 4.3

Lemma 4.2 implies that an optimal τ_1 must be contained in a compact set. We show that the objective function can be represented as a continuous function of τ_1 . Then, the optimization problem can be reduced as choosing an optimal τ_1 , which can be contained in a compact interval $[0, \tau^*]$.

Fix $\bar{q} < 1$, and consider an optimal solution from the classic problem for demand curve $D(q_0, \bar{q})$. Let \mathbf{q} and T_f be the optimal solution and the number of periods to complete the trading for the *classic problem*. Define $q_{T_f} = \bar{q}$. Next, we find the amount of the stock y_f which makes it optimal for the monopolist to make the first acceptable offer according to (4.16):

$$y_f = \sum_{t=1}^{T_f} (q_t - q_{t-1}) \beta^{-t}.$$

Then, choose τ_1 such that

$$e^{-b\tau_1} y = y_f.$$

We have to make it sure that τ_1 is well defined and $\tau_1 \geq 0$. This is the case if $\Delta > 0$ is sufficiently small.

Lemma C.1. $\forall \bar{q}, \forall y > 1, \exists \bar{\Delta} > 0$ such that $\forall \Delta \in (0, \bar{\Delta}), y_f < y$.

Proof. The optimal path has the property that the number of periods to complete the sales increases as \bar{q} increases. If $\bar{q} \leq q_f < 1$, then $T_f < \infty$ uniformly bounded. Moreover, the optimal path $\mathbf{q}(0, q_f, y)$ has the longest periods to complete the trading. Since the Coase conjecture holds for $\mathbf{q}(0, q_f, y)$, $\Delta T_f(0, q_f, y_0) \rightarrow 0$. Hence, once the monopolist begins to make an acceptable offers, the amount of real time to complete the sales converges to 0, if $\bar{q} \leq q_f < 1$. As a result, the amount of decay which occurs during the periods when the monopolist is making acceptable offers is very small. Thus, $\exists \bar{\Delta} > 0$ such that $\forall \Delta \in (0, \bar{\Delta}), y_f < y_0$. \square

Since y_f is a strictly increasing function of \bar{q} , the mapping from \bar{q} to τ_1 is one-to-one. Thus, for each τ_1 , we can find at most one \bar{q} which in turn determines $\mathbf{q}(0, \bar{q}, y)$ and T_f uniquely. Thus, the objective function

$$e^{-b\tau_1} \sum_{t=1}^{T_f} (q_t - q_{t-1}) \delta^{t-1}$$

changes continuously with respect to τ_1 , if $\{q_t\}$ is the optimal path associated with the classic problem with demand curve $D(0, \bar{q})$ and $\bar{q} \leq q_f < 1$. Since τ_1 is contained in a compact set, the objective function has a maximum. This proves Proposition 4.3.

APPENDIX D. PROOF OF PROPOSITION 4.5

Fix a state (q_0, q_f, y) and assume $q_0 = 0$ without loss of generality. Let p_1 be the initial offer according to the solution from (4.13). Consider $p' \neq p_1$. We need to construct a strategy profile for the monopolist, which remains optimal following p' .

Let us examine the case where $p' < p_1$. Choose $q' > q_0$ such that $p' = P(q', q_{T_f}, y)$. But, such q' may not exist if $p' < P(q_{T_f}, q_{T_f}, y)$. In such a case, choose $q' = 1 - p'$. Recall the construction process of the equilibrium. We select y_f so that $\beta^{T_1} y = y_f$, where the right hand side is a strictly increasing function of q_{T_f} . To highlight the parameters, let us write $y_f(q_0, \bar{q})$ in place of y_f . By construction of y_f , $\bar{q} = q_{T_f}$. Since $q' > q_0$,

$$y > y_f(q', q_{T_f}) + q'.$$

The accelerated sale requires less stock to meet the terminal condition.

Recall that $P(\cdot)$ is continuous and the graph of $P(\cdot)$ does not intersect with each other, if $\exists q'_{T_f}$ such that $p' = P(q_0, q'_{T_f}, y)$, then $q'_{T_f} > q_{T_f}$. Since y_f is a strictly increasing continuous function of q_{T_f} , $y = y_f(q_0, q_{T_f}) < y_f(q_0, q'_{T_f})$. Since $P(\cdot)$ is a continuous function of q_{T_f} , $\exists q'_{T_f}$ and $\exists q' > q_0$ such that

$$p' = P(q', q'_{T_f}, y)$$

and

$$y = y_f(q', q'_{T_f}) + (q' - q_0).$$

Since $q' > q_0$, any consumer to purchase at the equilibrium price will accept any $p' < p$.

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