

Toolboxes, Cards, and Individuality

March 13, 2002

The carpenter that built our house arrived each morning in a truck that contained several cabinets. In these cabinets were the tools of his trade: a drill, a nail gun, and various sized hammers, levels, saws, and screwdrivers. As important as the tools themselves were his skills as a carpenter – his ability to use the tools. One of the themes of this book will be that each of us regardless of our profession possesses our own set of tools that we can apply with varying levels of expertise. As we go through life, we acquire new tools and occasionally lose tools. Sometimes we use them correctly. Other times, we find ourselves pounding a nail with a cross cut saw.

Our personal differentiate us. Some of us know how to speak French. Others of us can crochet, throw horseshoes, or mince onions. These skill sets simultaneously define us, constrain us, and guide us. They determine who are friends are, what we eat, how we play, what careers we choose, how much money we make, how “smart” people think we are, and whether we are capable of having much fun. To lead fulfilling lives, we must choose them wisely, yet leave room for caprice.

Our personal toolboxes depend on both nature and nurture. We vary in our genetic ability to acquire skills. Some people can roll there r's effortlessly and speak fluent Spanish. Others, those with high concave palates, can spend hours saying “pot of tea” as fast as we can, yet never really engineer a roll. Our friends, family, and teachers also influence these tools. Growing up on a lake, not learning to swim or ice skate would be almost impossible. In some families, learning to play chess or cards is mandatory. Without even realizing it, these children will learn basic combinatorics and how to perform backward induction.

Due to genetics, social pressure, or interest level we differ in our capacity to possess skills, in the rates with which we can acquire skills, in our retention of those skills, in the rates at which we can apply them, and in our abilities to use them in combination. To say someone is “smart” is to say that they have facility in one or more of these areas. Some people may be quick to learn and apply skills, and they may be have enormous skill capacities. They may speak up first in class. Others may be slow to learn and slow to apply, but they may be better at retaining. Or they may be better at using skills in combination.

Or, they may be worse at everything. Anna may be able to hold twenty skills in her head, while Bob can only hold fifteen. Anna may learn skills faster, retain more

of them, and apply them more quickly. She may even be more adept at using them in combination. Does that mean she is “smarter” than Bob? In some weak sense, yes. But it does not mean that her intelligence subsumes Bob’s. Bob can and probably will possess a skill set that Anna does not. He may have more friends, score higher on standardized tests, and lead a more fulfilling life than Anna. It all comes down to how their skill sets mesh with the reality within which they find themselves. Given her natural advantages, Anna is likely to outperform Bob on many dimensions, but given that the set of skill sets is so large, a certain amount of luck enters the picture.

0.1 A Deck of Cards

The story of Anna and Bob can be more concretely understood through simple models. Imagine that each of fifty two cards in a deck represents a distinct skill. Anna can choose twenty cards, and Bob can choose fifteen. We want to ask three questions of this model Question 1: how many different skill sets can Anna choose? Question 2: how many different skill sets can Bob choose? and Question 3: how likely is it that Anna skill set equals Bob’s skill set plus more stuff? Or to phrase this last question more pointedly, how likely is it that anything Bob can do Anna can do better?

Anna can choose twenty cards. There are fifty cards she can choose first, fifty one she can choose second, and so on. So we just have to multiply fifty-two times fifty-one, times fifty, etc.. all the way down to thirty-three. However, we also need to take into account order if she chooses the ace of hearts and then the jack of spades, this is the same thing as choosing the jack of spades and then the ace of hearts. The number of ways the same twenty cards can be reordered equals twenty times nineteen times eighteen times . . . times one. Dividing the first number by the second, we obtain that the total number of unique skill sets Anna could acquire equals

$$\frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48 \cdot \dots \cdot 33}{20 \cdot 19 \cdot 18 \cdot 1}$$

This equals 125,994,627,894,135, or just shy of one hundred and twenty six trillion different skill sets. As something to chew on for a minute, you might want to recall that the best estimates of the total number of people who have lived on earth are on the order of 8 billion.

If we make a similar computation for Bob, we obtain that there are a mere 4,481,381,406,320, or roughly four and a half trillion skills sets for him to choose. Finally, to answer question three, how likely is it that Anna possesses every skill that Bob possesses all we have to do is choose an arbitrary skill set for Bob and compute the number of Anna’s skill sets that contain it. If we take Bob’s fifteen cards out of the deck there are thirty-seven left. Anna can choose five of these. There are

$$\frac{37 \cdot 36 \cdot 35 \cdot 34 \cdot 33}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

or 435,897 ways to do this. Fixing Bob’s skill set at some random collection of fifteen skills, of the 126 trillion skill sets that Anna might choose, fewer than a half million

of hers contain all of Bob's skills. If we divide the 126 trillion figure by the smaller number, we get that the odds are one in about 289,046,788. There is only a one in three hundred million chance that Anna possesses every skill that Bob does.

Now of course, givent that there are several billion people in the world, and if each of those people were choosing twenty cards, then the odds are that several of them would know everything Bob knows. But we should be careful here. The set of possible skills is far larger than fifty two. It is probably in the tens of thousands. Moreover, the number of skills an individual can possess is probably in the low thousands or at a minimum in the high hundreds. The deck of cards example was meant to be illustrative. The numbers of skill sets is incomprehensively large. Carl Sagan, who himself acquired quite an impressive tool kit, would be speechless.