# Economic Performance and Robustness

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## 1 Optimization and Differentiation

In this lecture, we consider three models that originated in economics that can help us understand implications of diversity. In that these are economic models much of the emphasis will be on benefits and costs, but as has been our tendency, we will interpret these models both within and outside their field of origin. The first model reveals a relationship between diversity and robustness. This model pertains to portfolio selection under uncertainty. Finance professors use this to show why risk averse investors should not pick the stocks with the highest expected return but instead should choose portfolios of stocks that have a diversity of performances across the possible states of the world. That is not to say that stock experts want to pick some stocks that will do well and some that will do poorly. Far from it. The idea is that not even the best stock, say Microsoft, always performs well, so investors should pick other stocks that will go up in value when Microsoft's price falls. We're going to interpret this model broadly and argue that the idea that diversification is a form of insurance applies to more than just selecting stocks. It applies to nations choosing economic policies and undergraduates selecting classes.

Our second model is one of the most famous models in economics. It formalizes the theory of comparative advantage. This model provides powerful logic against diversity. The model demonstrate that countries who can trade benefit from specialization - or diversity reduction. You can use the model to show why a more productive nation would want to trade with its less productive neighbors. Most people find this result counter-intuitive. The subtle intuition that drives this result is that relative efficiency not absolute efficiency is what matters. Any difference in relative efficiencies creates an incentive for specialization and trade. As a result of this trade, each nation's economy becomes less diverse. In our first pass at this model, we'll accept that conclusion, but later on in the lecture we'll question it.

Our third model combines specialization, one of the older ideas from economics, with one of the newer ones, learning by doing. This model will show how specialization increases productivity rates. We use this model to put some formal structure on Adam Smith's over told story of the pin factory.

Once we have understood these three models, we weave them together to describe a fundamental tension between diversity and specialization. We prefer diversification at the macro level because it reduces risk, and we want the micro level agents to specialize so that they will be more productive. This is what we have learned from the three models. This seems neat and tidy until we realize that there are more than two levels. Thinking globally, individual nations are the micro level agents and they should specialize, but at the national level, industries are the micro level agents. We wind up with a world in which each level wants to diversify but wants the level below it to specialize.

# 2 A Model of Portfolio Selection

We construct a simple model of portfolio selection in which there are a small number of states of the world and that these states influence the amount that investments pay.[1] As is standard, we assume that our investor is risk averse and that although she does not know what the state of the world will be, she has enough information and the cognitive capacity to construct a probability distribution over the possible outcomes.

We refer to an investor as risk averse, if she prefers a certain payoff to a lottery that has the same expected value but some risk associated with it. As an example, a risk averse person would prefer lottery $A$ over lottery $B$ and lottery $B$ over lotter $C$.

*Lottery A:* \$100

*Lottery B:* \$200 with probability one-half and \$0 with probability one-half.

*Lottery C:* \$400 with probability one-fourth and \$0 with probability three-fourths.

Risk aversion is a standard assumption in economics and for good reason. With moderate and large stakes, almost everyone has some aversion to risk. Only desperate people take significant portions of their savings and go play black jack. We have ample evidence that people are not risk averse in all settings. People bet on horse races and spend billions on lottery tickets. In each of these cases, you can make the case that people get what economists call "utility" from playing or from thinking about the possibility of winning. Therefore, the behavior would not be risk loving.[2] Further, people seem to exhibit less risk aversion when it comes to losses. Some psychologists even believe that most people are risk loving over losses, i.e. would rather have a 50-50 chance of losing \$200 than lose \$100 for sure, but risk averse over gains. Since we are talking about important decisions that pertain to gains, the risk aversion assumption

[1] Portfolio theory as taught in business schools assumes investments that have continuous distributions with correlated values. So, for example, two investments would each be a signed a distribution with a mean and a standard deviation, say normal with mean rate of return of 5% with a standard deviation of 3%, and the pair of investments would be assigned a correlation coefficient, $\beta$.

[2] I am not fond of this sort of ex post theorizing, but in this instance it has some purchase.

makes sense.

The model we construct has only three parts: the states of the world, the investments, and the probability assignments to the states of the world. The states of the world are a finite list of the events that are relevant to outcomes.

*Def'n: The set of* **states of the world** $\Omega = \{1, 2, ...\omega\}$.

Every investor considers all of the possible states of the world. However, we can extend this model to allow for the investors to have distinct perspectives or even interpretations of the states of the world. Differences in interpretations could cause people to have differing opinions about the success of particular investments, as could differing opinions about the probabilities of the various states. We address these possibilities in a few moments. For now, we assume that everyone had the same interpretation of the states of the world and also has the same beliefs about the probabilities of those states.

An investment is a vector of payoffs in each state of the world.

*Def'n: An* **investment** $\pi = (\pi_1, \pi_2, ...\pi_\omega)$ *where each* $\pi$ *is a real number.*

**Example:** Let the state of the world correspond to the weather $\{sunny, rainy, overcast\}$. Investing in an umbrella (UM) generates a payoff of one if it is sunny, twenty if it is rainy, and nothing if it is overcast. We can write its payoffs as $UM = (1, 20, 0)$.

To complete the model, we need to include probabilities of each state of the world occurring. We let $p_i$ denote the probability of state $i$ occurring. Each $p_i$ must be weakly positive and less than or equal to one. (You might ask why we would include a state that has probability zero. Hold onto that thought for a moment.)

*Def'n: A* **probability vector over states** $p = (p_1, p_2, ...p_\omega)$ *where* $p_i \geq 0$ *for all* $i$ *and* $\sum_{i=1}^{\omega} p_i = 1$.

Given a probability vector, we can compute the *expected value* of an investment.

*Def'n: The* **expected value** *of investment* $s$ *given the probability vector over states* $p$ *equals*

$$E[s \mid p] = \sum_{i=1}^{\omega} p_i \cdot \pi_i$$

Extending our earlier example, if sun, rain, and overcast weather are equally likely, then $p = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$, and the expected value of an umbrella equals seven:

$$E[UM \mid p] = \frac{1}{3} + \frac{20}{3} + \frac{0}{3} = 7$$

We add two more investments to show the value of diversity: sunglasses and a book. Their payoffs and associated expected values are as follows:

*Sunglasses (SG):* $SG = (18, 0, 0)$

$$E[SG \mid p] = \frac{18}{3} + \frac{0}{3} + \frac{0}{3} = 6$$

*Book (BK):* $BK = (0, 9, 9)$

$$E[BK \mid p] = \frac{0}{3} + \frac{9}{3} + \frac{9}{3} = 6$$

Notice that $UM$ has the highest expected value of the three investments. To show how diversity matters, we need to consider the behavior of an investor confronted with these three possible investments. Suppose that each of these investments costs \$5, and that a risk averse investor has \$15 dollars to spend. How should she spend it? Consider the following three portfolios:

| | |
|---|---|
| *UM Portfolio* | $UM = 2$, $SG = 0$, $BK = 0$ |
| *One - Two Portfolio* | $UM = 0$, $SG = 1$, $BK = 2$ |
| *All Three Portfolio* | $UM = 0$, $SG = 1$, $BK = 2$ |

We can compute the payoff vectors for each of these three portfolios as follows: The UM Portfolio's payoff vector equals three times the payoff vector of the UM investment, or $(3, 60, 0)$. The One - Two Portfolio has a payoff vector that equals the sum of the SG investment's payoff vector $(18, 0, 0)$ and two times the BK investment's payoff vector $(0, 18, 18)$, which is $(18, 18, 18)$. Finally, the All Three Portfolio has a payoff vector that is a some of the three investment's individual payoff vectors: $(21, 29, 9)$[3].

In evaluating these portfolios, we can consider various measures. We have already talked about *expected value*. The problem with using expected value is that it does not take into account risk aversion. Therefore, we might also calculate the *worst* payoff. In this instance, all three states of the world have probabilities of $\frac{1}{3}$, so the worst payoff is relevant. In general, the problem with looking at the worst payoff is that it could be of low probability and we do not want to assume that people condition their behavior on low probability events.

More important might be the *median* payoff. The median payoff is the payoff such that half the time the investment pays off at least this well and half of the time it does at most this well. In the table below, we compare all three portfolios on each of these three measures.

| **Portfolio** | **Payoff Vector** | **Expected Value** | **Worst** | **Median** |
|---|---|---|---|---|
| *UM* | $(3, 60, 0)$ | 21 | 0 | 3 |
| *One -Two* | $(18, 18, 18)$ | 18 | 18 | 18 |
| *All Three* | $(19, 29, 9)$ | 19 | 9 | 19 |

[3] $(21, 29, 9) = (3, 20, 0) + (18, 0, 0) + (0, 9, 9)$

When looking at the portfolios in this way, the One-Two Portfolio stands out as the best choice for someone risk averse even though it has the lowest expected value. The One-Two Portfolio offers perfect insurance against uncertainty. No matter what the state of the world, the payoff will always be 18. The other two portfolios fare poorly on overcast days. The UM Portfolio pays nothing and the All Three Portfolio only pays out 9.

This example helps us to think about diversity differently. If we think of diversity at the level of investments, then the All Three Portfolio is the most diverse as it is the only portfolio that contains all three investments. However, if we interpret diversity at the level of payoffs in the possible states of the world, the One-Two Portfolio is the most diverse.[4]

This example contains several ideas. We see how a diversity of investments can be preferred to the best single investment by a risk averse investor. This suggests a link between diversity and robustness. We explore that idea at length in a later lecture. We also see that where we measure diversity matters. Greater diversity of investments need not imply more a more diverse portfolio. How the individual investments payoffs vary according to the state of the world.

## 2.1 Other applications

So far this has been a story about investments, but the intuition applies more generally. Recall our toolbox model of skills. Consider a college student deciding which courses to take. Taking a course adds tools to the toolbox and hones existing tools. Students often have difficulty choosing courses because they do not know how they will lead their lives: what career they will choose, where they will leave, what will their passions be, etc.. In light of this uncertainty, diversification makes sense. If there is some possibility that the state of the world will have you in Germany - then risk aversion says that you should learn German. The notion that a well lived life requires a diversity of skills and understandings partially underpins the idea of a core curriculum.[5]

Similarly, if someone asks you to bring something home for dinner and if you are not certain of the person's preferences, you may choose to diversify over the possible states of the world and buy all sorts of things. You could go to a deli and get a salad, and some bread cheese and meat. That way regardless of whether the person is a vegetarian, a carnivore, lactose intolerant, or even gluten intolerant, you are covered.

The second lesson also applies. Diversity at the level of states of the world need not imply diversity at the level of choices. Instead of being asked to bring dinner, suppose that a guest you have never met calls ahead and says that he spilled coffee on his pants and asks if you could pick up some pants for him. Here, it might make

[4]By this we mean that if we were to apply any of our measures of diversity from the first lecture, this portfolio would be the most diverse.

[5]The other foundational assumption is that we need a shared culture, but if all we needed was a shared culture, we could all be biologists.

sense to buy blue jeans as they would be considered acceptable by almost anyone.

## 2.2 Differences of Opinion and Making Horse Races

In our portfolio model, people would trade in order to maintain diversified portfolios as information about the state of the world changes. Such trades would not account for the overall level of trade that we see in the stock market. People trade for many reasons, among them what are often called "differences of opinion." This is one of those catchall phrases that requires unpacking. Our use of models can help us differentiate between two types of differences of opinion. In our model, the "difference" must either come from two people having different probability values over the states of the world, or from having distinct interpretations and therefore not having the same states of the world. We won't dig into this here, but it's something to think about.

# 3 Comparative Advantage

In our second model, we consider two countries, Advancia and Backwardistan whose citizens must decide whether to trade with one another. Each country possesses technology to make two things: bread and automobiles. The citizens in each country want to consume equal amounts of automobiles and bread. The table below displays the production technologies in the two countries if all workers were allocated to each technology. For example, the workers in Advancia could make 160 automobiles in one day if they made no bread.

| **Output Per Day** | | |
|---|---|---|
| **Country** | **Autos** | **Bread** |
| Advancia | 160 | 240 |
| Backwardistan | 20 | 80 |

In the absence of trade, each country divides up the days of the week so as to produce equal amounts of both automobiles and bread. Advancia workers spend three days a week making automobiles and two days a week making bread for a total output of 480 units of each commodity. In Backwardistan, the workers only manage 80 units of each and owing to their technological shortcomings, must spend four days making cars.

*No Trade:*

| **Advancia** | | | |
|---|---|---|---|
| **Product** | **Days** | **Rate** | **Bread** |
| Autos | 3 | 160 | **480** |
| Bread | 2 | 240 | **480** |

| Backwardistan | | | |
|---|---|---|---|
| **Product** | **Days** | **Rate** | **Bread** |
| Autos | 4 | 20 | **80** |
| Bread | 1 | 80 | **80** |

Even though Backwardistan is less effective than Advancia at producing each commodity, both countries benefit from trade. As we discussed above. These may seem counter intuitive. The key to understanding how this can be is to consider relative and not absolute productivity. Backwardistan workers are relatively a lot better at baking bread than they are at producing cars. They are one third as efficient in the bread shop but only one eighth as efficient in the factories. In other words, Backwardistan has a *comparative advantage* in bread making. The optimal solution then is to have Backwardistan only make bread, and for Advancia to maintain a diverse economy.

### *Comparative Advantage*

| Advancia | | | |
|---|---|---|---|
| **Product** | **Days** | **Rate** | **Bread** |
| Autos | 4 | 160 | **640** |
| Bread | 1 | 240 | **240** |

| Backwardistan | | | |
|---|---|---|---|
| **Product** | **Days** | **Rate** | **Bread** |
| Autos | 0 | 20 | **0** |
| Bread | 5 | 80 | **400** |

With trade, the total production in both economies equals 640 units of both automobiles and bread. Here is just one possible division of those spoils.

*Advancia:* 540 units of each commodity, a 12.5% increase

*Backwardistan:* 100 units of each commodity, a 25% increase

This model suggests that diversification at the level of a country is not a good thing, the countries benefit by focusing on those technologies for which they have a comparative advantage. If the goal of a country is to maximize consumption in the current period, then this is probably true, but as we shall see, more realistic country level objects argue against specialization. Before turning to that topic, we first consider another model which even more pointedly shows the benefits from specialization.

# 4 The Coconut Factory

Our last model demonstrates the benefits from specialization in a dynamic context. In the model, there are two people Robinson and Friday. They live on an island. Robinson and Friday live on coconuts and fish. Each had large appetite and each likes to spend time relaxing on the beach and wading in the crystal blue waters. Robinson and Friday are both intelligent people. The more they do something, the better they perform.

Business professors and psychologists would call this improvement a learning curve. Learning is pervasive in society. Every day, billions of people improve at their daily tasks. Some of this learning comes from new skill acquisition, sometimes people interpret problems or situations in a new way, and sometimes new technologies allow for improvements. Learning rates tend to be decreasing over time. A worker at a new job may get 10% bettter the first year, improve by another 5% the second year, and then improve by only 1% the third year. This has nothing to do with a decrease in effort. It has everything to do with the fact that the easy improvements are found first. Eventually, improving becomes difficult.

Assume that Robinson and Friday each wants to eat as much fish and as many coconuts as possible, and that each wants to work exactly ten hours per day. If initially each of them can pick one coconut per hour and catch one fish per hour, then their total production equals ten coconuts and ten fish. It does not matter how the allocate their time between fishing and who picking coconuts so long as they collectively spend equal amounts of time at each task.

We want to show how with learning by doing, specialization improves performance. Assume a simple learning rate of twenty four over $n$ percent increases where $n$ is the number of years spent entirely on that job. If Robinson splits his time between the two tasks, he improves at one half of that rate. The first table shows Robinson's (or Friday's) learning rates and productivity if he splits his time equally between the two tasks.

Productivity Without Specialization

| **Year** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** |
|---|---|---|---|---|---|---|---|---|
| Learning Rate | 12 | 12 | 6 | 6 | 4 | 4 | 3 | 3 |
| Output | 11.2 | 12.5 | 13.3 | 14.1 | 14.7 | 15.2 | 15.7 | 16.2 |

The table above shows that if they divide the tasks equally, after eight years, they will be 62% more efficient. Each will be eating 62% more with no increase in work effort. Suppose instead that they specialize. This enables them to learn more quickly: they get more productive faster.

Productivity With Specialization

| Year | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Learning Rate | 24 | 12 | 8 | 6 | 4.8 | 4 | 3.43 | 3 |
| Output | 12.4 | 13.9 | 15.0 | 15.9 | 16.7 | 17.3 | 17.9 | 18.5 |

We now see that if they specialize that productivity will grow by 85% over the eight years. This is a substantial increase over the 62% productivity gain that occurred had they not specialized.[6]

# 5 Comparative Advantage and Learning by Doing

We now want to combine our latter two models to make an even stronger case for specialization. We assume that the workers in the two countries learn while they produce. As in our third model, this implies that daily output of a commodity increases with the amount of time spent producing that commodity. To keep the mathematics simple, we will assume that the daily output of a commodity increases yearly at a rate equal to the number of days per week spent in production on that commodity. This means that a country that spends three days a week making cars will be able to make three more cars per day at the end of each year.[7]

Let's return to our two economies and see what has happened after twenty years of no trade. Assume first that they had continued the same production schedule.

*No Trade:* After 20 Years

| **Advancia** | | | |
|---|---|---|---|
| **Product** | **Days** | **Rate** | **Bread** |
| Autos | 3 | 160 + 3*20 = 220 | **660** |
| Bread | 2 | 240 + 2*20 = 280 | **560** |

| **Backwardistan** | | | |
|---|---|---|---|
| **Product** | **Days** | **Rate** | **Bread** |
| Autos | 4 | 20+4*20 = 100 | **400** |
| Bread | 1 | 80+1*20 = 100 | **100** |

These numbers don't quite make sense because both countries now make more automobiles than they need relative to bread. Over time, each country would have

[6]Notice that the learning rate for the specialist is higher in each period. This is true in general for this type of learning rate. If the learning rate after $n$ periods of work equals $\frac{k}{n}$ for someone working full time and $\frac{k}{2M}$, where $M$ begins at one and increments by one every two periods, for someone working half time, then the learning rate for the the specialist exceeds the learning rate for the person who diversifies.

[7]Note that this formulation implies a diminishing learning rate because an increase of three cars becomes smaller as a percentage of total output as total output increases.

put more time into making bread, and as a result become more productive at it. To correct for this, here are some more reasonable production numbers after twenty years.

*New Production:*

| **Advancia** | | | |
|---|---|---|---|
| **Product** | **Days** | **Rate** | **Bread** |
| Autos | 2.9 | 210 | **609** |
| Bread | 2.1 | 290 | **609** |

| **Backwardistan** | | | |
|---|---|---|---|
| **Product** | **Days** | **Rate** | **Bread** |
| Autos | 3 | 80 | **240** |
| Bread | 2 | 120 | **240** |

The total production of each commodity equals 849 units under the no trade scenario. Alternatively, let's suppose that the two countries have been trading for twenty years. As a benchmark, assume that they maintain their previous production schedules.

*Specialization and Trade:* After 20 years

| **Advancia** | | | |
|---|---|---|---|
| **Product** | **Days** | **Rate** | **Bread** |
| Autos | 4 | 160 + 4*20 = 240 | **960** |
| Bread | 1 | 240 + 1*20 = 260 | **260** |

| **Backwardistan** | | | |
|---|---|---|---|
| **Product** | **Days** | **Rate** | **Bread** |
| Autos | 0 | 20 | **0** |
| Bread | 5 | 80+5*20 = 180 | **900** |

This arrangement does not produce enough cars so it is reasonable to assume that at some point during the twenty years, Advancia would have begun to put even more effort into automobile production. Their output table might look as follows:

| **Advancia** | | | |
|---|---|---|---|
| **Product** | **Days** | **Rate** | **Bread** |
| Autos | 4.3 | 250 | **1075** |
| Bread | 0.7 | 250 | **175** |

Now the total production equals 1075 units of each commodity. This is over a 25% improvement. The gain has two sources: specialization and learning by doing. Both

benefit because Backwardistan no longer produces autos, something it is terrible at doing and because over the twenty year period Backwardistan got much better at making bread.

# 6 Diversification and Robustness

We have just made a pretty compelling case against diversification and for specialization. Our argument does not consider our first model of portfolio selection. Should and individual country be concerned with the expected value of the global economy or with it's own performance under the various states of the world that might occur? Probably the latter. So, to take this lecture full circle, let's suppose that in some states of the world trading relations halt. This could be due to a war, epidemic, religious differences or even political posturing or misunderstandings.

We can compare the countries in isolation after twenty years of the two scenarios. We arrive at very different conclusions.

*Initial Economies:*

| **Advancia** | | | |
|---|---|---|---|
| **Product** | **Days** | **Rate** | **Bread** |
| Autos | 3 | 160 | **480** |
| Bread | 2 | 240 | **480** |

| **Backwardistan** | | | |
|---|---|---|---|
| **Product** | **Days** | **Rate** | **Bread** |
| Autos | 4 | 20 | **80** |
| Bread | 1 | 80 | **80** |

*No Trade:* After 20 Years
*New Production:*

| **Advancia** | | | |
|---|---|---|---|
| **Product** | **Days** | **Rate** | **Bread** |
| Autos | 2.9 | 210 | **609** |
| Bread | 2.1 | 290 | **609** |

| **Backwardistan** | | | |
|---|---|---|---|
| **Product** | **Days** | **Rate** | **Bread** |
| Autos | 3 | 80 | **240** |
| Bread | 2 | 120 | **240** |

*Specialization and Trade: Now Isolated* After 20 years

| Advancia | | | |
|---|---|---|---|
| **Product** | **Days** | **Rate** | **Bread** |
| Autos | 2.5 | 250 | **625** |
| Bread | 2.5 | 250 | **625** |

| Backwardistan | | | |
|---|---|---|---|
| **Product** | **Days** | **Rate** | **Bread** |
| Autos | 4.5 | 20 | **90** |
| Bread | 0.5 | 180 | **90** |

We see that Backwardistan remains backward. They have only increased output by ten units in each commodity.

In addition to robustness to trade wars, an economy should be robust to technological shocks. If a shock occurs, then all producers of a good immediately can immediately produce more. Non producers do not experience the shock. Suppose that a shock occurs that increases production of automobiles by 100 per day.

*No Trade Economies after shock:*

| Advancia | | | |
|---|---|---|---|
| **Product** | **Days** | **Rate** | **Bread** |
| Autos | 2.6 | 240 | **624** |
| Bread | 2.4 | 260 | **624** |

| Backwardistan | | | |
|---|---|---|---|
| **Product** | **Days** | **Rate** | **Bread** |
| Autos | 2 | 120 | **240** |
| Bread | 3 | 80 | **240** |

*Trading Economies after shock:*

| Advancia | | | |
|---|---|---|---|
| **Product** | **Days** | **Rate** | **Bread** |
| Autos | 3.2 | 260 | **832** |
| Bread | 1.8 | 240 | **432** |

500x 1600

| Backwardistan | | | |
|---|---|---|---|
| **Product** | **Days** | **Rate** | **Bread** |
| Autos | 0 | 20 | **0** |
| Bread | 5 | 80 | **400** |

Again, we see that Backwardistan is worse off because of trading.

# 7 Conclusions and Homework.

In sum, we see this trade off for a desire for diversification at the level of decision making but a strong desire for specialization at lower levels. Globalists would like countries to specialize. Presidents prefer diversified economies but would like regions and states to specialize. Governor want diverse state economies with regional specializations. Mayors want local diversification, and so on..

## 7.1 Homework

Suppose that you are the benevolent dictator in charge of the world economy. Choose which levels you would encourage or demand specialization and which levels you would encourage or demand diversification. Put your answers in the chart below and then write one page defending your decisions.

| **Level** | **Diversify (D) or Specialize (S)** |
|---|---|
| Global | |
| International | |
| National | |
| Regional | |
| Local | |
| Organizational | |
| Team | |
| Individual | |