Diversity and Coordinating Interactions

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1 I'm Okay, You're Okay

Friends, peers, family members, competitors, and public figures influence our actions, beliefs, and behavior. This general phenomenon is called many things - peer pressure, herd behavior, and jumping on the band wagon to name just a few. These tendencies to coordinate with one another are not always a force for good. Negative interactions contribute to several of our pressing social problems including crime, drug use, and poor student performance. In each of these cases there is a tendency for conformity. This is true in systems that don't involve people as well. Systems of magnets and crystals create forces to locally align. We shall refer to interactions in which there is a force such that each person, chemical agent, magnet, crystal or other physical entity tends to match the state or action of another as a coordinating interaction.

In this lecture, we explore how coordinating interactions contribute to diversity levels. The BIG ideas in this lecture pertain to how interactions can create diversity across populations that were initially similar, and on how these interactions tend to homogenize attributes and actions within populations. This lecture requires slow careful thinking. In each of the three substantive areas, we reinterpret the same model to make both points: *coordinating interactions create diversity across populations and dampen within population diversity.* In other words, the model predicts that Germans will be similar to one another but be quite different from Canadians. This lecture contains multiple interwoven themes. The substantive topics include physical, ecological, and social systems which we analyze with a variety of formal approaches including spin glasses, differential equations, and game theory.

We analyze models that settle into equilibria and then compare the equilibria. Keep two images in mind: an egg carton and a bowl. If you drop a marble into an egg carton, it can wind up in any of twelve spots; whereas, if you drop that same marble into a bowl, it always ends up at the same spot. Most systems of people are more like egg cartons – many outcomes are possible.[1]

[1] I should add one caveat, almost all systems have an odd number of equilibria and so you should properly think of an egg carton with spaces for thirteen eggs.

2 Physics: A One Dimensional Spin Glass Model

Our first model comes from physics and is commonly referred to as a spin glass. A spin glass consists of a collection of *sites* each site has an initial *spin*. These spins can be positive, we denote this by (S) or negative (N). We can think of S and N as representing South and North and think of these as magnets that must choose one of two orientations. A spin glass is a discrete time dynamical system. This mean that there are discrete time steps. In each time step each site can change its spin. Most spin glass models are two dimensional. Below is an example of an initial configuration:

S	S	N	N
S	S	S	S
S	N	S	N
N	S	N	S

Each site updates its spin according to the spins of its neighbors. In the diagram below we highlight one site and its neighbors to the North, South, East, and West - the so called the Von Neumann neighbors.

		N	
	S	S	S
		S	

One rule for updating in a spin glass model is that each site matches the spin of a majority of its neighbors. If no majority exists, the site maintains the same spin. This system can cycle, but in most cases it settles into an equilibrium. We compare these equilibrium states to see how much diversity this system can generate.

At this point, lest we get too bogged down in physics, we can give this model many other interpretations. Consider these two social science applications in which we view the sites as people. First, S can denote standing and N can denote sitting and this can be a model of a standing ovation. Alternatively, people who smoke or study can be assigned S's and those that do not given N's. The dynamics then capture peer pressure.[2]

To make things simple, we consider a one dimensional spin glass model with only five sites.

S	S	S	S	N

[2] The peer pressure is a bit too homogeneous to be realistic. Most people do not use majority rule in deciding whether to smoke.

We assume that a site's neighborhood consists of the two sites to its left and right. We make the two sites at the ends neighbors of one another so that each site has exactly two neighbors.[3]

2.1 Diversity Creation

We first show how this model can explain diversity. We begin with a random collection of spins and then compute the possible equilibrium configurations that result. There are thirty two possible initial site configurations each is written in the first column of the table below. In the subsequent columns, we denote the state of the spin glass over time according to the updating rule: *change your spin only if both of your neighbors have the opposite spin.*

[3]This one dimensional spin glass model is a special case of a one dimensional cellular automata. In fact, it is a famous cellular automata known as the voter model because each site "votes" on what it thinks is the most popular spin.

Initial State	time 1	time 2
SSSSS	SSSSS	SSSSS
SSSSN	SSSSS	SSSSS
SSSNS	SSSSS	SSSSS
SSSNN	SSSNN	SSSNN
SSNSS	SSSSS	SSSSS
SSNSN	SSSNS	SSSSS
SSNNS	SSNNS	SSNNS
SSNNN	SSNNN	SSNNN
SNSSS	SSSSS	SSSSS
SNSSN	NSSSS	SSSSS
SNSNS	SSNSS	SSSSS
SNSNN	NSNNN	NNNNN
SNNSS	SNNSS	SNNSS
SNNSN	NSNNS	NNNNN
SNNNS	SNNNS	SNNNS
SNNNN	NNNNN	NNNNN
NNNNN	NNNNN	NNNNN
NNNNS	NNNNN	NNNNN
NNNSN	NNNNN	NNNNN
NNNSS	NNNSS	NNNSS
NNSNN	NNNNN	NNNNN
NNSNS	NNNSN	NNNNN
NNSSN	NNSSN	NNSSN
NNSSS	NNSSS	NNSSS
NSNNN	NNNNN	NNNNN
NSNNS	SNNNN	NNNNN
NSNSN	NNSNN	NNNNN
NSNSS	SNSSS	SSSSS
NSSNN	NSSNN	NSSNN
NSSNS	SNSSN	SSSSS
NSSSN	NSSSN	NSSSN
NSSSS	SSSSS	SSSSS

By the second time period every one of the configurations has reached an equilibrium. Of the thirty two original configurations, twelve are possible equilibria. For each of these twelve equilibrium configurations, we can compute the number of configurations that attain that equilibrium. This is known as the *basin size*. We can also compute the number of sites who have one neighbor that differs. This is known as the *frustration*. The table below gives the frustration and the basin size for each of the twelve equilibria.

Equil. State	Frustration	Basin Size
SSSSS	0	11
NNNNN	0	11
NNSSS	4	1
NSSSN	4	1
SSSNN	4	1
SSNNS	4	1
SNNSS	4	1
NNNSS	4	1
NNSSN	4	1
NSSNN	4	1
SSNNN	4	1
SNNNS	4	1

Notice that the equilibrium configurations contain patches of S's and N's. If we had a larger number of sites, the patchiness would be even more pronounced. These patches are important. Suppose we think of the initial state as someone's predisposition to smoke. This is something that we might estimate by knowing their age, socio-economic status, race, education, and so on. These patches imply that it will be important to know the predispositions of someone's friends as well. The same logic apples to the problem of predicting school performance and criminal behavior at the level of individual. That is true.[4] Without going too far out on a limb, we can say that this model partially explains why these remain problems. It's hard to predict how to solve them because the interaction effects muddle our analyses.

This table also demonstrates that a simple system of physical interactions can generate a diversity of equilibrium configurations, but that each of these configurations itself is less diverse than a random configuration. Thus, we see our first example of within group homogeneity and across group heterogeneity.

2.2 Trembles and Diversity Reduction

We can extend this model to show how interactions may further dampen within group diversity. Alter the previous model by assuming that every other time period with some small probability *exactly* one of the sites trembles and switches its spin.[5] This does not effect the two sites with the lowest frustration. Consider the configuration SSSSS. If the middle site switches its spin, the new configuration, SSNSS, returns to the original configuration in the next period. In contrast, if we take any of the other configurations, say SSNNN, it is not stable with respect to some spin switches.

[4] Glazer, Sacerdote, and Schienkman use a model almost identical to this to explain variations in crime rates in the U.S.

[5] By making the perturbations every other time period, we allow the system to react prior to another perturbation.

Suppose SSNNN becomes NSNNN. In the next period, it becomes NNNNN. SSNNN could also become SSSNN or SSNNS. Both are equilibria and both can become SSSSS through a single tremble.

It is not hard to show that given these trembles that in the long run the configuration converge to either SSSSS or NNNNN, and that each of these is equally likely. Why does this matter? This says that if there are trembles in the system, we cannot have diversity at the level of the sites. All sites are either S or N. If there were a population of spin glasses, then we could have diversity at the level of configurations, but we would not have diversity within any configuration.

2.2.1 Spring boarding the Idea

We construct these models to help organize our thinking. This is more than just an exercise in creating mathematical metaphors - not that those are all bad! In thinking about applying this model to the worlds of people and idea, we want to ask whether the assumptions make sense. The spin glass model has several problematic assumptions. It has a specific and in most cases unrealistic contact structure. If we were to consider social interactions, we would not arrange people linearly. Nor would we assume that each person has exactly two friends.

Nevertheless, this model can help us understand levels of diversity. Once again, think of this as a model of smoking. The model suggests that if there were no coordination and no trembles, then we would see either "patchiness" or complete homogeneity. For example, we might see that women in sororities smoke more than we would expect were we to predict on an individual level.

As another example, again consider each site as a person but think of the spin as the person's political beliefs. To keep the model simple, assume people are either liberal, conservative, or moderate. Assume a richer interaction structure in which people have varying number of friends perhaps with different strengths. Allow some people to have higher thresholds for switching their beliefs. Some people may flow with the majority, others may be more hard headed. Thad Browne has studied this and found that these hard headed people make the system more predictable at the local level. He finds that a hard headed conservative is likely to end up surrounded by other conservatives. Also assume that it is harder to jump all the way from liberal to conservative than it is to jump from conservative to liberal. Even with all these bells and whistles added, this new model should still generate "patches" or what might be called spatial correlation in political preferences.[6] There are many other problem including the fact the relationships are fixed and the interactions take place on only one dimension. We'll get to those problems later.

What would we learn from such a model? Here's one conjecture. For a given distribution of social connections, it might well be that the equilibria are biased toward

[6]Sprague and others have studied this, and with some important caveats have found this type of relationship among preferences. The exception being that people and their best friends do not have very similar beliefs.

four types of configurations. One in which everyone is liberal. One in which everyone is conservative, one in which everyone is moderate, and one in which the numbers of liberals, conservatives, and moderates are approximately equal. This would imply that political opinion on an issue is likely to either be diverse or homogeneous as a result of the interactions. That is an intuition worth contemplating. Do coordinating interactions create a bias toward homogeneity within populations and heterogeneity across them.

3 An Ecological Model

This brief section is included to show that one of the core ecological models supports our previous intuition. In this model, we assume two species denote their levels by x and y. The percentage change in the population of the x's in each period equals $a - x - by$ and the percentage change in the population of y's in each period equals $a - y - bx$. In these equations both a and b are positive. These functional forms capture an interaction in which each type crowds out itself. The more x's, the slower the x's grow. They also capture a competitive relationship between the x's and the y's. The more x's the worse the y's do.

Example: Suppose that $a = 100$, $b = \frac{1}{2}$, $x = 40$ and $y = 80$. In the next period, x will grow by 20% $(100 - 40 - 40)$. So there will be 48 x's. The y's will stay fixed at 80 $(100 - 80 - 20)$.

It can be shown that if $b > 1$, then either the y's or the x's take over the entire population. In ecology and biology texts, this is called *competitive exclusion*. If $b < 1$, then equal numbers of y's and x's exist in equilibrium. In this model, we see that coordinating interactions can lead to within population homogeneity or diversity. But if we look more closely at the interaction terms, we see that we get homogeneity within a population if b which is the pressure for other type to take over the population is large. We only get diversity (equal numbers of x's and y's) when the interaction between the two types is small. Further, this model supports diversity across populations owing to the multiplicity of equilibria.

4 Game Theory and Diversity

Early game theory was described as the study of interactions between strategic actors, but in the past twenty years, game theory has also been used to study populations of evolving actors who need not be strategic. Therefore, it may be more accurate to say that game theory is the study of environments in which the payoffs to one actor depend on the actions of others. If this seems to you like it should always be true, you're right. The question is, when is the effect large enough to go through the hassle of including it in our thinking.

Game theory can be used to predict outcomes, explain behavior, and design institutions. Here, we employ game theory to show how interactions can create and dampen diversity. We begin with a simple game known as the Battle of the Sexes. The Battle of the Sexes game has two players: Boris and Aliza. Each must choose whether to go to the opera or to the theatre. Boris prefers the theatre to the opera, and Aliza prefers the opera to the theatre. Both, however, would sacrifice their own desires to be with the other person.

We can capture this strategic interaction in a game by ascribing utilities to each action and creating a game matrix. The columns of the matrix correspond to Boris's actions, and the rows correspond to Aliza's actions. The numbers correspond to the utility (or happiness, or payoff) that Aliza and Boris get from that outcome. For example, if Aliza chooses "Opera" and Boris chooses "Opera", then Aliza gets utility of 5 and Boris gets utility of 3.

The Battle of the Sexes

Boris

	Actions	**Opera**	**Theatre**
Aliza	**Opera**	5,3	2,2
	Theatre	0,0	3,5

A pair of strategies create an equilibrium if given the strategy of the other player, neither player wants to change his or her strategy. It is a stable point relative to the actions of the players. For example, both players choosing to go to the opera is an equilibrium. If Boris were to switch and go to the theatre, his utility falls from 3 to 2. Similarly, if Aliza were to go to the theatre instead, her utility drops from 5 all the way to zero. In contrast, if Aliza choose to go to the opera and Boris chooses to go to the theatre, it is not an equilibrium. Each gets a utility of 2. And, if Boris were to choose opera instead, he'd get a utility of 3.[7]

Imagine now that Boris and Aliza both have identical twins. Let's call them Bobo and Alice. Let's further suppose that Bobo and Alice have the same utilities in this game as Boris and Alice. (In making this latter assumption, we're ignoring the previous lecture about why even with identical genes and similar life experiences people may have different preferences.) Given these assumptions, it is possible that Boris and Aliza could choose to go to the opera and Bobo and Alice would choose to go to the theatre. Thus, "identical" pairs of people would choose different outcomes: *within population homogeneity, across population heterogeneity.*

[7]Both players choosing the opera and both players choosing the theatre are the only equilibria in pure strategies. A player's strategy is pure if it is does not randomize between actions, i.e. if it takes one action with probability one. There is a third equilibrium to this game (remember our comment about the number of equilibria being odd) in which both players randomize between the two actions.

This rather modest example has some rather profound implications. To see why, let's consider an even simpler game, a variant of something called the pure coordination game. Here we refer to the players as Rowland and Colleen. In this game, players have to choose whether to bow or shake when meeting one another. If both players shake hands, each gets a utility of a, which we assume to be greater than zero. If both bow, each gets a utility of one. If they take opposite actions, each gets a utility of zero.

Greetings

Colleen

	Actions	**Shake**	**Bow**
Rowland	**Shake**	a,a	0,0
	Bow	0,0	1,1

As in the Battle of the Sexes game, there are two pure strategy equilibria to the Greetings game. In one, both players bow. In the other, both players shake hands.[8] To predict which equilibria is most likely, we would need to know exactly how big a is. To secure our logic, we consider an extreme example: the Maui-Des Moines Game, where each player must choose either Maui or Des Moines. If both choose Maui, they both get a week long vacation in Maui. If both choose, Des Moines, they both get a week long vacation in Des Moines. If they choose distinct locations, neither gets anything.

Maui-Des Moines

Colleen

	Actions	**Maui**	**Des Moines**
Rowland	**Maui**	100,100	0,0
	Des Moines	0,0	1,1

Given these payoffs, we would expect the Maui equilibrium to occur far more often than the Des Moines equilibrium.

In the Greetings game, we could go through the exercise of trying to estimate a or we could try to infer a from behavior. The former boils down to asking whether bowing generates higher utility than shaking hands. Bowing has some advantages. You don't have to touch the other person. You can vary the speed, depth, and style of the bow. But, bowing can also strain your back. Shaking hands has the advantage that you can vary duration and strength and type of shake. Shakes can be vigorous or slow moving. When shaking hands, your hand should be clean and dry, but not

[8]This game also has a mixed strategy, making a total of three equilibria.

too dry. Through introspection, neither bowing or shaking hands would seem to have much of an advantage over the other, so a is probably close to one.

Observation would support this intuition. A survey would show that in most Western countries people shake hands, and in most Eastern countries people bow. One explanation for this could be differences in preferences, that for some reason Easterners genetically get higher utility from bowing, but this seems rather forced. A more compelling explanation is that a is close to one and independent of genetics and that it is pure chance that Easterners bow and Westerners shake hands. One reason that we do not see much variation within geographic regions of the globe, such as say "those crazy bowing Portuguese" is that due to trade, war, and migrations, similar patterns of behavior were likely to emerge.

Before leaving the Greetings game, let's suppose for the sake of argument that scientists at the Center for Disease Control discovered that shaking hands is the primary cause for the spread of flu viruses. With this new information, the value of the parameter a would fall and the hand shaking equilibrium would be less attractive. Would we the stop shaking hands?

4.1 Lock-in

The fact that the hand shaking equilibrium becomes less attractive does not mean that people would abandon it. In fact, if the equilibrium is played by a large number of people, it could become locked-in even though it is inefficient. In practice, the Greetings game differs from the Maui-Des Moines game and the Battle of the Sexes game in that each person plays it with many other people each day. You might play the Greetings game with one hundred people daily. If you and a friend were the only two people who played the Greetings game, then as soon as a fell below one, the two of you could chat and decide "based on the medical evidence, we should bow." However, having that conversation with everyone you are likely to meet would be an onerous task. Someone would have to take out an ad or there would have to be a public signal.[9]

Consider the case of our use of the English systems of weights and measures. Two hundred years ago the utilities from using the metric and English systems might have been about the same, but now we would get much higher utility were we to use the metric system. However, given that everyone else in the United States uses the English systems, it is difficult to engineer a switch. We have been able to make the switch on some things like food products but we have been less successful on things like speed limits, or heights and weights, etc....

4.2 Games Theory and Culture

Our analysis so far looks at single game. We saw that identical people could choose to play identical games differently owing to the multiplicity of equilibria. We now

[9]See Michael Chwe's book on common knowledge.

want to make the plausible assumption that we play more than one game each day – that we play multiple games. We also assume that people benefit informationally, computationally, psychologically, and socially from using similar strategies across these games. Examples help to clarify each of these separate effects.

Informational Benefits: Suppose a professor has two offices in which he can meet people. He might use one as a place to meet students and the other as a place to meet with faculty. If instead, the professor were to have a different rule for each person: meet Mary in office A, Steven in office B, etc.., he would have to remember a great deal more information.

Computational Benefits: Suppose that whenever you are asked to bid in an auction setting, you bid 75% of your true value in the first round. This requires less computation than than thinking through what the possible auction rules might be, and it may work reasonably well in almost any environment.

Psychological Benefits: In playing a game many times, one way to enforce cooperation is to punish players who deviate. There are two common punishing strategies:minimal punishing and maximal punishing. The latter are often called Grim Trigger strategies. In real life, these take the form of "if you ever lie to me, we are no longer friends." If you alternate these strategies depending upon the game it could be psychologically more taxing. In addition to the informational costs of having to remember which games you punish at which level, your capriciousness may be internally taxing.

Social Benefits: Just as being consistent makes it easier on ourselves, it also makes it easier on other people.[10] If other people know that I always react violently to anyone who disagrees with me, they can better condition their own actions. If instead, my actions vary depending on the situation, my friends have to make calculated guesses.

4.3 Life Experiences and Culture

We can use these ideas to flesh out part of the "different life experiences make us diverse" story. Define different life experiences as having been in distinct sets of strategic situations, which we can model as games. Or alternatively, we might have been in the same situations but seen them in different orders or with different frequencies. In light of this we might accumulate different sets of tools (heuristics). Owing to our distinct tool sets, we may (in some cases) not respond the same way to an identical situation. Thus, our diverse histories of "games" generate a diversity of repertoires: we learn to either bow or shake hands. Coordinating interaction games create pressures toward within group homogeneity, and therefore across group heterogeneity.

This is not a complete story, life experiences contain more than games. The

[10]What is easy for me to know about myself and what is easy for others to know are not at all the same thing. Psychological consistency and individual consistency differ.

skills, behaviors, tools, and even the ideas that we carry around in our heads get shaped and chosen in myriad ways. That said, we should not dismiss the insight that coordination games of the type we consider here cause people who interact frequently to have similar skills, behaviors, and ideas. This may cause us to act more similar than we might otherwise. However, the similarity of our behavioral repertoires hinges upon us playing similar games with similar frequencies. It is unlikely that we will be alike if they are not.[11].

If we want to extend this argument to the level of ethnicity, culture, or nationality, we need not take many cognitive leaps. If people living in Japan or Switzerland experience a different set of situations than we do, they may acquire a set of tools suited for those situations. What we think of as cultural differences might be explained as having appropriate tool kits for our different lives. Moreover, this connects to the situational interpretation of behavior. Our actions

Homework: Take a particular institution or environment and describe how it shapes the tools that are used. For example, you could analyze why people in small towns are nicer than people in big cities? Why are people from one dorm different from another? What happens to people who go to Michigan State or Haverford? Why do people in subdivisions with uncontrolled intersections interact more frequently or less frequently? Architecture, geography, and religion are all avenues to pursue here.

[11]This is easily satisfied. If we play twenty coordination games and each has two equilibria, then there are over one million possible collections of equilibria.